Note: The following list of problems is much longer than the number of problems that will appear on the actual midterm. Some problems may also turn out to be more difficult than the problems you'll see on the exam. On average, however, these problems should be fairly similar in difficulty to the exam problems, and they cover most of the material that you should review.

1. Give examples of linear systems in two variables with (a) no solutions, (b) one solutions, (c) infinitely many solutions.

2. Consider the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$.

   (a) Give an example of a linear system whose coefficient matrix if A.

   (b) Give an example of a linear system whose augmented matrix is A.

   (c) Describe all solution to the system in (b). How many solutions are there?

3. Given the definitions of the following: (a) *row operation*, (b) *echelon form*, (c) *reduced echelon form*, (d) *leading entry*, (e) *pivot position*, (f) *pivot column*, (g) *basic variable*, (h) *free variable*.

4. Suppose your phone number is 12345678. Form the 3×3 matrix

$$A = \begin{bmatrix} x & 1 & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}.$$

   (You might try this problem with your own phone number instead.)

   (a) Substitute an arbitrary value for x, and then compute the reduced echelon form of A.

   (b) Find another value for x which results in A having a different reduced echelon form.

   (c) Describe the possible values of $\texttt{RREF}(A)$ as a function of x.

5. Given the definitions of (a) *linear combination*, (b) *span*, and (c) *linear independence* of a set of vectors.

6. Determine if the columns of the matrices

$$A = \begin{bmatrix} 0 & -8 & 5 \\ 3 & -7 & 4 \\ -1 & 5 & -4 \\ 1 & -3 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -4 & -3 & 0 \\ 0 & -1 & 4 \\ 1 & 0 & 3 \\ 5 & 4 & 6 \end{bmatrix}$$

   are linearly independent.

7. Compute AB^T and BA^T, with A and B defined as in the previous problem.

8. Do the columns of A or B span $\mathbb{R}^4$?

   Do the columns of A^T or B^T span $\mathbb{R}^3$?

9. Suppose $f : \mathbb{R}^n \to \mathbb{R}^m$ is a function. Say what it means for f to be (a) *linear*, (b) *one-to-one*, (c) *onto*, (d) *invertible*.

10. Suppose $T : \mathbb{R}^n \to \mathbb{R}^m$ is onto and linear. What are the possible values for $n - m$?

11. Suppose $T : \mathbb{R}^n \to \mathbb{R}^m$ is one-to-one and linear. What are the possible values for $n - m$?

12. Determine if the matrix
$$A = \begin{bmatrix} 0 & -8 & 5 \\ 3 & -7 & 4 \\ -1 & 5 & -4 \end{bmatrix}$$
is invertible. If it is, compute its inverse.

13. Consider the martrix
$$A = \begin{bmatrix} a & b & 0 & 0 & 0 \\ c & d & 0 & 0 & 0 \\ 0 & 0 & e & 0 & 0 \\ 0 & 0 & 0 & f & g \\ 0 & 0 & 0 & h & i \end{bmatrix}.$$
What is $\det A$? When is A invertible? Assuming A invertible, given a formula for A^{-1}.

14. Given the definition of the following (a) *subspace* of $\mathbb{R}^n$, (b) *basis* of a subspace, (c) *dimension* of a subspace.

15. Let A be an $m \times n$ matrix. Given the definition of the following (a) the *nullspace* of A, (b) the *column space* of A, and (c) the *rank* of A.

16. Suppose A is an $m \times n$ matrix. What are the possible values for $\operatorname{rank} A$? What are the possible values of $\dim \operatorname{Nul} A$?

17. Find a basis for the nullspace of
$$A = \begin{bmatrix} 0 & -8 & 5 \\ 3 & -7 & 4 \\ -1 & 5 & -4 \\ 1 & -3 & 2 \end{bmatrix}.$$

18. Find a basis for the column space of A^T, with A defined as in the previous problem.

19. Is the function
$$T\left(\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}\right) = \det \begin{bmatrix} 1 & v_1 & 3 \\ 4 & v_2 & 6 \\ 7 & v_3 & 9 \end{bmatrix}$$
a linear transformation $\mathbb{R}^3 \to \mathbb{R}$? If it is, compute its standard matrix.

20. Suppose you have two matrices A and B of the same size. How would you construct a matrix C whose nullspace is the intersection of $\operatorname{Nul} A$ and $\operatorname{Nul} B$?

21. Suppose you have two matrices A and B of the same size. How would you construct a matrix C whose column space contains both $\operatorname{Col} A$ and $\operatorname{Col} B$?

22. Compute the determinant of
$$A = \begin{bmatrix} 0 & x & 0 & 0 \\ y & 0 & 0 & 0 \\ 0 & 0 & z & 0 \\ 0 & 0 & 0 & w \end{bmatrix}.$$

23. Does there exist a 2×2 matrix A with all entries in $\mathbb{R}$ such that $A^2 v = -v$ for all $v \in \mathbb{R}^2$? If not, say why. If there is, give an example. (Recall that $A^2 = AA$ for a square matrix.)