MIDTERM - MATH 2121, FALL 2019.

Name:

Student ID:

Email:

Lecture: L1 (Wed/Fri) L2 (Mon/Wed)

Tutorial: T1A T1B T1C T2A T2B T2C

Problem #	Max points possible	Actual score
1	10	
2	10	
3	20	
4	10	
5	10	
6	10	
7	10	
Total	80	

Complete these exercises and clearly write up your solutions.

Problem 1. (10 points) Find all solutions to the linear system

$$\begin{aligned} x_1 - x_2 - 6x_3 &= 10 \\ 2x_2 + 7x_3 &= -10 \\ x_1 + x_2 + x_3 &= 0 \end{aligned}$$

Solution:

Problem 2. (10 points)

Find all values of a such that $\left\{ \begin{bmatrix} 1 \\ a \end{bmatrix}, \begin{bmatrix} a+2 \\ a+6 \end{bmatrix} \right\}$ is linearly independent in $\mathbb{R}^2$.

Solution:

Problem 3. (20 points) Indicate which of the following is TRUE or FALSE.

(1) An invertible matrix can have more than one echelon form.
(2) Suppose U and V are subspaces of $\mathbb{R}^2$. If $\dim U < \dim V$ then $U \subset V$.
(3) Suppose U and V are subspaces of $\mathbb{R}^3$. If $\dim U < \dim V$ then $U \subset V$.
(4) If $T : \mathbb{R}^n \to \mathbb{R}^m$ is linear and onto then $n \geq m$.
(5) Four vectors in $\mathbb{R}^3$ can be linearly independent if they are all nonzero.
(6) If $\det A = \pm 1$, then A must be a permutation matrix.
(7) If $a + d + g = b + e + h = c + f + i = 0$ then $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ is not invertible.
(8) Suppose A and B are matrices such that AB is defined. If AB is invertible then A and B are either both invertible or both not invertible.
(9) The inverse of a permutation matrix is the same as its transpose.
(10) If two rows of a square matrix A are the same then $\det A = 0$.

Each part will be graded as follows: 0 points for a wrong or missing answer, 2 points for the correct answer. Explanations are not required for answers.

Solution:

(1)	TRUE	FALSE
(2)	TRUE	FALSE
(3)	TRUE	FALSE
(4)	TRUE	FALSE
(5)	TRUE	FALSE
(6)	TRUE	FALSE
(7)	TRUE	FALSE
(8)	TRUE	FALSE
(9)	TRUE	FALSE
(10)	TRUE	FALSE

Problem 4. (10 points)

Suppose $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation with

$$T\left(\begin{bmatrix} 2 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix} 5 \\ 8 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ 9 \end{bmatrix}.$$

Find the standard matrix of T.
In other words, find a 2×2 matrix A such that $T(v) = Av$ for all $v \in \mathbb{R}^2$.

Solution:

Problem 5. (10 points) Consider the matrix

$$A = \begin{bmatrix} 5 & 5 & 6 & 6 & 7 \\ 4 & 0 & 4 & 4 & 0 \\ 3 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 5 & 6 & 7 & 1 & 8 \end{bmatrix}.$$

(a) Compute A^{-1} or explain why A is not invertible.

(b) Compute $\det A$.

Solution:

Problem 6. (10 points) Let m and n be positive integers.

Suppose $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation with standard matrix A.
Recall that this means that A is a matrix such that $T(v) = Av$ for all $v \in \mathbb{R}^n$.

(a) How many rows does A have? How many columns does A have?

(b) If T is one-to-one, then what is the dimension of the column space of A? Explain your answer to receive full credit.

(c) If T is onto, then what is the dimension of the null space of A? Explain your answer to receive full credit.

Problem 7. (10 points)

Find a basis for $\mathbb{R}^4$ that includes the vectors $u = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix}$ and $v = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 2 \end{bmatrix}$.

In other words, find vectors $w, x \in \mathbb{R}^4$ such that u, v, w, x is a basis for $\mathbb{R}^4$.
Justify your answer to receive full credit.

Solution: