Mathematical Excalibur

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Olympiad Corner

Below are the problems of the 2012 IMO Team Selection Test 1 from Saudi Arabia.

Problem 1. In triangle ABC, points D and E lie on sides BC and AC respectively such that $AD\perp BC$ and $DE\perp AC$. The circumcircle of triangle ABD meets segment BE at point F (other than B). Ray AF meets segment DE at point P. Prove that $DP/PE = CD/DB$.

Problem 2. In an $n\times n$ board, the numbers 0 through n^2-1 are written so that the number in row i and column j is equal to $(i-1)+n(j-1)$ where $1\le i,j \le n$. Suppose we select n different cells of the board, where no two cells are in the same row or column. Find the maximum possible product of the numbers in the n cells.

Problem 3. Let $\mathbb{Q}$ be the set of rational numbers. Find all functions $f:\mathbb{Q}\to\mathbb{Q}$ such that for all rational numbers x,y,

$$f(f(x)+xf(y)) = x+f(x)y.$$

Problem 4. Find all pairs of prime numbers p, q such that $p^2-p-1 = q^3$.

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On-line:
http://www.math.ust.hk/mathematical_excalibur/

The editors welcome contributions from all teachers and students. With your submission, please include your name, address, school, email, telephone and fax numbers (if available). Electronic submissions, especially in MS Word, are encouraged. The deadline for receiving material for the next issue is ***November 20, 2012***.

For individual subscription for the next five issues for the 09-10 academic year, send us five stamped self-addressed envelopes. Send all correspondence to:

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IMO 2012 (Member Perspective)

Andy Loo

This year's International Mathematical Olympiad (IMO) has been of considerable significance to Hong Kong. At the 1997 IMO held in Mar del Plata, Argentina, shortly after our official transfer of sovereignty, the Hong Kong delegation accomplished the special mission of elucidating Article 149 of its *Basic Law* in light of Annex I of the *Sino-British Joint Declaration*, thereby consolidating the legitimacy of its participation in the IMO. This July, following the 15th anniversary of the establishment of the Special Administrative Region, this annual event returns to Argentina, in exactly the same city as last time's. In addition to battling in the examination hall, the Hong Kong team was endowed with the invigorating task of bringing the IMO to Hong Kong again in 2016.

Joined by 542 young brains from 99 countries, the Hong Kong team comprised the following personnel: Dr. Leung Tat Wing (leader), Mr. Leung Chit Wan (deputy leader) and the team members were Kevin Lau Chun Ting (St. Paul's Co-educational College), Andy Loo (St. Paul's Co-educational College), Albert Li Yau Wing (Ying Wah College), Jimmy Chow Chi Hong (Bishop Hall Jubilee School), Kung Man Kit (SKH Lam Woo Memorial Secondary School) and Alice Wong Sze Nga (Diocesan Girls' School).

This contest bestows certain personal touch upon me, for it not only marks my unprecedented landing on the continent of South America, but is also my first and, in all probability, my last IMO, an ultimate platform for me to display my years of Mathematical Olympiad endeavor in my high school career. Having represented Hong Kong at both the International Physics Olympiad (IPhO) and the IMO is a great responsibility which I feel extremely grateful to have had the unique chance to shoulder.

<u>**July 7 and 8**</u> Our flights from Hong Kong to Frankfurt and from Frankfurt to Buenos Aires, each over 12 hours long, were predominantly occupied by sleep and math exercises, considering the disappointing fact that our planes turned out to be two of the very few Boeing-74748 models of Lufthansa that lack in-flight entertainment systems. Our amazement at a German flight attendant, who spoke more than fluent Mandarin Chinese, as well as a cozy conversation with a Slovakian neighbor, highlighted the otherwise uneventful journey.

We arrived at the Argentinean capital city early in the morning of July 8 (in winter!), and, after being transported to the domestic airport, employed a time-consuming conglomeration of *Google Translate* effort and sign language to manage to purchase a couple of SIM cards at a tiny store, where the shopkeeper knew literally no English. A Maradona-like bus driver kindly offering us a free ride, we embarked on a tour around the city and enjoyed a beef-dominated meal before returning to the airport in the late afternoon to catch our flight to Mar del Plata, on which I, being absolutely exhausted, slept from the first to the last minute.

<u>**July 9**</u> The major event of this day was the Opening Ceremony. It was held in the Radio City. I met British team member Josh Lam and congratulated him on his mother's recent promotion to Chief Secretary of Hong Kong. If I were to describe the entire ceremony in one word it would definitely be "Spanish". Almost all the speeches were delivered in Spanish, albeit accompanied by English interpretation. To most, the more exciting parts of the ceremony included the IMO anthem, the parade of nations and the distant waves from the leaders, who were forbidden to communicate with us before the contest as they took part in problem selection.

July 10 On this first day of the contest, we had 3 problems to solve in 4.5 hours. Because questions could only be raised in the first 30 minutes, I had to understand all the problems quickly.

Problem 1 Given triangle ABC the point J is the center of the excircle opposite the vertex A. This excircle is tangent to the side BC at M, and to the lines AB and AC at K and L, respectively. The lines LM and BJ meet at F, and the lines KM and CJ meet at G. Let S be the point of intersection of the lines AF and BC, and let T be the point of intersection of the lines AG and BC. Prove that M is the midpoint of ST.

I decided to use my favorite method – complex numbers. Indeed, denote the complex number representing each point by the corresponding small letter. Setting j=0 and m=1, I found s=2k/(k+l) and t=2l/(k+l) after a straightforward computation, and the result followed.

Problem 2 Let $n \geq 3$ be an integer, and let $a_2,a_3,\ldots,a_n$ be positive real numbers such that $a_2a_3\cdots a_n = 1$. Prove that

$$(1+a_2)^2(1+a_3)^3\cdots(1+a_n)^n > n^n.$$

Inequalities were once among the hottest topics on the IMO but totally disappeared in the last three years due to the rising popularity of brute force techniques, e.g. Muirhead's inequality and Schur's inequality. But my firm belief in the revival of inequalities has never been shaken, and instead was only strengthened by Problem 5 of APMO 2012. Consequently I had done appreciable preparation in this area before the Olympiad.

In IMO history, this problem was quite unique. For one, it is an n-variable inequality. For the other, it has no equality case. Both features are unparalleled according to my memory.

I spent about an hour attempting to solve the problem using induction or analysis, with no avail. In despair, I took logarithm and applied Jensen's inequality by appealing to concavity of the log function. Miraculously, it gave precisely the inequality in the problem! After checking that equality case cannot satisfy the condition $a_2a_3\cdots a_n$ =1, I was basically done.

Then on a second thought, I realized that I could actually convert my proof into a logarithm-free one that involves the AM-GM inequality only. So I rewrote my solution in this new form and marked the original as an alternative solution. It turned out that Alice was also able to solve this problem with the AM-GM inequality.

Problem 3 The *liar's guessing game* is a game played between two players A and B. The rules of the game depend on two positive integers k and n which are known to both players.

At the start of the game A chooses integers x and N with $1 \leq x \leq N$. Player A keeps x secret, and truthfully tells N to B. Player B now tries to obtain information about x by asking player A questions as follows: each question consists of B specifying an arbitrary set S of positive integers (possibly one specified in some previous question), and asking A whether x belongs to S. Player B may ask as many such questions as he wishes. After each question, player A must immediately answer it with *yes* or *no*, but is allowed to lie as many times as she wants; the only restriction is that, among any k+1 consecutive answers, at least one answer must be truthful.

After B has asked as many questions as he wants, he must specify a set X of at most n positive integers. If x belongs to X, then B wins; otherwise, he loses. Prove that:

1. If $n \geq 2^k$, then B can guarantee a win.
2. For all sufficiently large k, there exists an integer $n \geq 1.99^k$ such that B cannot guarantee a win.

This problem was not only long, but also terribly difficult. In the end, only 8 contestants managed to solve it. Despite my effort, the only thing I was able to do was proving the k = 1 case in Part 1, with the hope of getting slim partial credits.

Finally Day 1 of the contest was over. Our team aced Problem 1. As for Problem 2, Alice and I should be able to get 7's while Albert's partial analytic solution would be subject to vigorous debate. Kit also finished the k = 1 case in Part 1 of Problem 3. Overall I was satisfied with my performance on Day 1.

July 11 The six IMO problems are usually partitioned into the four categories (algebra, combinatorics, geometry and number theory) in the fashion of {1,5}, {2,4}, {3} and {6} (up to permutation). Judging from this pattern I would face an easy algebraic problem, an intermediate geometric problem and a hard number theoretic problem on Day 2. I figured that I would plausibly get a Gold medal for solving two of them, a Silver medal for one and a Bronze medal for none. My strategy was to guarantee Problem 4 and then aim to get Problem 5 by hook or by crook.

To my astonishment, Problem 4 was much more involved than I had expected. On the other hand I felt I could do Problem 5 with analytic tools:

Problem 5 Let ABC be a triangle with $\angle BCA$ = 90°, and let D be the foot of the altitude from C. Let X be a point in the interior of the segment CD. Let K be the point on the segment AX such that BK=BC. Let L be the point on the segment BX such that AL=AC. Let M be the point of intersection of AL and BK. Show that MK=ML.

I proceeded to do coordinate geometry, only to find out I was doomed after almost one hour. The reason was as follows. The expressions were quadratic in nature (as lengths took part in the formulation of the problem), leading to the prevalence of square roots. (As a side note, this also deterred me from using complex numbers, where one may have difficulty in selecting the correct roots of the quadratic equations.)

As the old Chinese saying goes, one should "drop his cleaver and become a Buddha (放下屠刀，立地成佛)". I decided to abandon Problem 5 for a moment and to reconsider Problem 4:

Problem 4 Find all functions $f: Z \to Z$ such that, for all integers a, b, c that satisfy $a + b + c = 0$, the following equality holds:

$$\begin{aligned} &f(a)^2+f(b)^2+f(c)^2 \\ &=2f(a)f(b)+2f(b)f(c)+2f(c)f(a). \end{aligned}$$

(Here Z denotes the set of integers.)

This was a problem with unusual answers. It took me quite a while to write up a tidy solution and to ensure that no point could sneak away from my hands. Thus it was 2.5 hours into Day 2. I still had Problems 5 and 6 left.

Problem 6 Find all positive integers n for which there exist non-negative integers $a_1,a_2,\ldots,a_n$ such that

$$\frac{1}{2^{a_1}}+\frac{1}{2^{a_2}}+\ldots+\frac{1}{2^{a_n}}=\frac{1}{3^{a_1}}+\frac{2}{3^{a_2}}+\ldots+\frac{n}{3^{a_n}}=1.$$

I quickly determined that Problem 6 was hopeless. Turning to Problem 5 again, I spent all the remaining time expanding everything. I was finally able to convince myself that my proof was complete.

(continued on page 4)

Problem Corner

We welcome readers to submit their solutions to the problems posed below for publication consideration. The solutions should be preceded by the solver's name, home (or email) address and school affiliation. Please send submissions to *Dr. Kin Y. Li, Department of Mathematics, The Hong Kong University of Science & Technology, Clear Water Bay, Kowloon, Hong Kong*. The deadline for sending solutions is ***November 20, 2012.***

Problem 401. Suppose all faces of a convex polyhedron are parallelograms. Can it have exactly 2012 faces? Please provide an explanation to your answer.

Problem 402. Let S be a 30 element subset of $\{1,2,\ldots,2012\}$ such that every pair of elements in S are relatively prime. Prove that at least half of the elements of S are prime numbers.

Problem 403. On the coordinate plane, 1000 points are randomly chosen. Prove that there exists a way of coloring each of the points either red or blue (but not both) so that on every line parallel to the x-axis or y-axis, the number of red points minus the number of blue points is equal to −1, 0 or 1.

Problem 404. Let I be the incenter of acute $\triangle ABC$. Let Γ be a circle with center I that lies inside $\triangle ABC$. D, E, F are the intersection points of circle Γ with the perpendicular rays from I to sides BC, CA, AB respectively. Prove that lines AD, BE, CF are concurrent.

Problem 405. Determine all functions f,g: $(0,+\infty) \to (0,+\infty)$ such that for all positive number x, we have

$$f(g(x)) = \frac{x}{xf(x)-2} \quad and \quad g(f(x)) = \frac{x}{xg(x)-2}.$$

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Solutions

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Problem 396. Determine (with proof) all functions f: $\mathbb{R}\to\mathbb{R}$ such that for all real numbers x and y, we have

$$f(x^2+xy+f(y)) = (f(x))^2 + xf(y) + y.$$

Solution. **AN-anduud Problem Solving Group** (Ulaanbaatar, Mongolia), **CHEUNG Ka Wai** (Munsang College (Hong Kong Island)), **CHEUNG Wai Lam** (Queen Elizabeth School), **Dusan DROBNJAK** (Mathematical Grammar School, Belgrade, Serbia), **Kevin LAU** (St. Paul's Co-educational College, S.4), **Simon LEE** (Carmel Alison Lam Foundation Secondary School), **Mohammad Reza SATOURI** (Persian Gulf University, Bushehr, Iran) and **Maksim STOKIĆ** (Mathematical Grammar School, Belgrade, Serbia).

Call the required equation (*). For x=0, we get $f(f(y))=y+f(0)^2$ for all y. Call this (**). The right side may be any real number, hence f is surjective. By (**), $y = f(f(y)) - f(0)^2$. If $f(y) = f(y')$, then the last equation implies $y=y'$, i.e. f is injective.

Putting $x=-y$ in (*), we get $f(f(y)) = (f(-y))^2-yf(y)+y$ for all y. Call this (***).

Now f surjective implies there exists z such that $f(z)=0$. Let $x=y=z$, then (*) yields $f(2z^2)=z$. Putting $(x,y)=(0,2z^2)$ in (*), we get $0=2z^2+f(0)^2$. Then z=0 and $f(0)$=0. So (**) reduces to $f(f(y))=y$ for all y. Putting y = 0 in (*), since $f(0)$ = 0, we get $f(x^2) = (f(x))^2$. The last two sentences reduce (***) to $y = (f(y))^2-yf(y)+y$. This simplifies to $f(y)$ = 0 or $f(y) = y$ for every y. Since f is injective and $f(0)$ = 0, we get $f(y) = y$ for all y. Conversely, a quick check shows $f(y) = y$ for all y satisfies (*).

Other commended solvers: **Tobi MOEKTIJONO** (National University of Singapore).

Problem 397. Suppose in some set of 133 distinct positive integers, there are at least 799 pairs of relatively prime integers. Prove that there exist a,b,c,d in the set such that $\gcd(a,b) = \gcd(b,c) = \gcd(c,d) = \gcd(d,a) = 1$.

Solution. **CHEUNG Ka Wai** (Munsang College (Hong Kong Island)), **Dusan DROBNJAK** (Mathematical Grammar School, Belgrade, Serbia), **Kevin LAU** (St. Paul's Co-educational College, S.4), **Simon LEE** (Carmel Alison Lam Foundation Secondary School), **Andy LOO** (Princeton University), **Tobi MOEKTIJONO** (National University of Singapore) and **Maksim STOKIĆ** (Mathematical Grammar School, Belgrade, Serbia).

Let $S=\{n_1, n_2,\ldots,n_{133}\}$ be the set of these 133 positive integers. From i=1 to 133, let X_i be the set of all n_k in S such that $k\neq i$ and $\gcd(n_i,n_k)$=1. Denote by $|X|$ the number of elements in set X. For $k\neq i$, $\gcd(n_i,n_k)$=1 implies $n_i\in X_k$ and $n_k\in X_i$. Then $N = |X_1| + |X_2| + \cdots + |X_{133}| \geq 2\times799 = 1598$.

Define $f(x) = x(x-1)/2$. In a set X with j elements, there are exactly $j(j-1)/2 = f(|X|)$ pairs of distinct elements. Since $f(x)$ is concave on $\mathbb{R}$, by Jensen's inequality,

$$\sum_{i=1}^{133} f(|X_i|) \geq 133 f\left(\frac{N}{133}\right) \geq 133 f\left(\frac{1598}{133}\right)$$

$$> 133 f(12) = f(133) = f(|S|).$$

Since every pair of distinct element in X_i is also a pair of distinct element in S, the inequality above implies in counting pairs of distinct elements in the X_i's, there are repetitions, i.e. there are X_i, X_k with $i\neq k$ sharing a common pair of distinct elements a,c. Let $b=n_i$ and $d=n_k$. Then a,b,c,d satisfy $\gcd(a,b) = \gcd(b,c) = \gcd(c,d) = \gcd(d,a) = 1$.

Problem 398. Let k be positive integer and m an odd integer. Show that there exists a positive integer n for which the number n^n-m is divisible by 2^k.

Solution. **AN-anduud Problem Solving Group** (Ulaanbaatar, Mongolia), **Dusan DROBNJAK** (Mathematical Grammar School, Belgrade, Serbia), **KWAN Chung Hang** (Sir Ellis Kadoorie Secondary School (West Kowloon)), **Kevin LAU** (St. Paul's Co-educational College, S.3), **Simon LEE** (Carmel Alison Lam Foundation Secondary School), **Andy LOO** (Princeton University), **Tobi MOEKTIJONO** (National University of Singapore) and **Maksim STOKIĆ** (Mathematical Grammar School, Belgrade, Serbia).

For k=1, let n=1. Suppose it is true for case k (i.e. there exists n such that $2^k \mid n^n-m$). Now m odd implies n odd. For case k+1, if $2^{k+1} \mid n^n-m$, then the same n works for k+1. Otherwise, $n^n-m=2^kl$ for some odd integer l. Let $v=2^k$. By binomial theorem,

$$\begin{aligned}(n+v)^{n+v} &= n^{n+v}+(n+v)n^{n+v-1}v+v^2x \\ &= n^{n+v}+vn^{n+v}+v^2y\end{aligned}$$

for some integers x,y. By Euler's theorem, since n is odd and $\varphi(2^{k+1})=2^k$,

$$n^v = n^{2^k} \equiv 1 \pmod{2^{k+1}}.$$

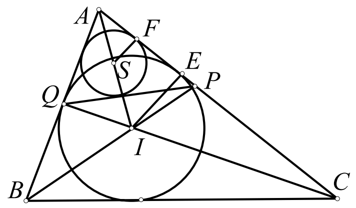
Since $l+n^n$ is even, we have

$$\begin{aligned}(n+v)^{n+v} &= n^{n+v}+vn^{n+v}+v^2y \\ &\equiv n^n+2^kn^n = m+2^k(l+n^n) \\ &\equiv m \pmod{2^{k+1}}.\end{aligned}$$

So $n+v$ works for k+1.

Problem 399. Let ABC be a triangle for which $\angle BAC=60°$. Let P be the point of intersection of the bisector of $\angle ABC$ and the side AC. Let Q be the point of intersection of the bisector of $\angle ACB$ and the side AB. Let r_1 and r_2 be the radii of the incircles of triangles ABC and APQ respectively. Find the radius of the circumcircle of triangle APQ in terms of r_1 and r_2 with proof.

Solution. **Dusan DROBNJAK** (Mathematical Grammar School, Belgrade, Serbia), **Kevin LAU** (St. Paul's Co-educational College, S.4), **Andy LOO** (Princeton University), **MANOLOUDIS Apostolos** (4° Lyk. Korydallos, Piraeus, Greece), **Tobi MOEKTIJONO** (National University of Singapore) and **Maksim STOKIC** (Mathematical Grammar School, Belgrade, Serbia).



Let I and S be the incenters of $\triangle ABC$ and $\triangle APQ$ respectively. (Note A,S,I are on the bisector of $\angle BAC$.) Now $\angle PIQ = \angle CIB = 180^\circ - (\angle CBI + \angle BCI)$ $= 180^\circ - ½(\angle CBA + \angle BCA) = 120^\circ$ using $\angle BAC = 60^\circ$. So $APIQ$ is cyclic.

Applying sine law to ΔAPI, we get $IP/(\sin\angle IAP) = 2R$. So $R = IP$. By a well-known property of incenter, we have $IP=IS$ (see *vol.11*, *no.2*, *p.1* of <u>*Math Excal*</u>.). Let the incircles of ΔABC and ΔAPQ touch AC at E and F respectively. Then $R=IP=IS = AI-AS$ $= IE/\sin 30^\circ - SF/\sin 30^\circ = 2r_1 - 2r_2$.

Other commended solvers: **AN-anduud Problem Solving Group** (Ulaanbaatar, Mongolia), **Ioan Viorel CODREANU** (Secondary School Satulung, Maramure, Romania), **Simon LEE** (Carmel Alison Lam Foundation Sec. School) and **Mihai STOENESCU** (Bischwiller, France).

Problem 400. Determine (with proof) all the polynomials $P(x)$ with real coefficients such that for every rational number r, the equation $P(x) = r$ has a rational solution.

Solution. **Tobi MOEKTIJONO** (National University of Singapore), **Maksim STOKIC** (Mathematical Grammar School, Belgrade, Serbia) and **TAM Ka Yu** (MIT).

We will show $P(x)$ satisfies the desired condition if and only if $P(x)=ax+b$, where $a,b \in \mathbb{Q}$ and $a \neq 0$. For the *if*-part, $P(x) = r \in \mathbb{Q}$ implies $x = (r-b)/a \in \mathbb{Q}$.

Conversely, let $P(x)$ satisfy the desired condition and let $n=\deg P$. For each $r = 0,1,\ldots, n$, let $P(x_r)=r$ for some $x_r \in \mathbb{Q}$. By the Lagrange interpolation formula,

$$P(x) = \sum_{r=0}^{n}\left(r \prod_{0\le r\le n, r\ne i} \frac{x-x_i}{x_r-x_i} \right).$$

Expanding the right side, we see $P(x)$ has rational coefficients.

Letting M be the product of the denominators, we see $Q(x)=MP(x)$ has integer coefficients. Let k be the leading coefficient of $Q(x)$ and c be the constant term of $P(x)$. Let p_1, p_2, p_3, … be the sequence of prime numbers. Let $P(x)=c+p_i/M$ has solution $t_i \in \mathbb{Q}$. Then $Q(x)-(cM+p_i)$ has k as the leading coefficient and $-p_i$ as constant term. Now $Q(t_i)=0$, which implies $t_i=1/d_i$ or p_i/d_i for some (not necessarily positive) divisor of k. Since $P(t_i)$'s are distinct, so the t_i's are distinct. Hence, $t_i=1/d_i$ for at most as many time as the number of divisors of k. So there must exist a divisor d of k such that there are infinitely many times $t_i=p_i/d$. This imply that $P(x)-(c+dx/M)=0$ has infinitely many solutions. So the left side is the zero polynomial. Then $P(x)=ax+b$ with $a=d/M\neq 0$ and $b=c$ rational.

Other commended solvers: **Simon LEE** (Carmel Alison Lam Foundation Secondary School).

IMO 2012 (Member Perspective)

(continued from page 2)

The arrival of Dr. Leung stirred up much happiness after the contest. We reported on how we did. Albert and Jimmy shone on Day 2, solving Problems 4 and 5. Kit was also comfortable with Problem 4 while Kevin had some technical troubles in one particular case. Nobody achieved anything substantial on Problem 6.

We celebrated that evening at a Chinese restaurant. It was especially memorable that our deputy leader raised a couplet (對聯), which he regarded as an open puzzle for millenniums (千古絕對):

望江樓，望江流
望江樓上望江流
江樓千古，江流千古

It took me nearly an hour to come up with a so-so solution:

觀雨亭，觀雨停
觀雨亭下觀雨停
雨亭四方，雨停四方

<u>**July 12**</u> It was the contestants' turn to have fun and the leaders' turn to work hard. At night, Dr. Leung briefed us on the progress of the first day of coordination. In addition to our previous expectations, Albert pocketed one point for proving the necessary condition on Problem 6. Regretfully, Kit lost one point on Problem 4 for not having verified the feasibility of the functions obtained. Dr. Leung had refused to sign Alice's and Kevin's scores on Problem 4 in order to bargain later.

<u>**July 13**</u> The marking scheme stipulated that any solution of Problem 5 with coordinate geometry would score a 0 if not a 7. Despite our leaders' relentless effort, the coordinators were able to detect a fatal error of mine. So my Problem 5 was destined to be a 0.

On another note, Dr. Leung succeeded in getting 1 point for Alice on Problem 4, which in his words was "an achievement". Kevin's Problem 4 was finalized with a score of 4.

<u>**July 14**</u> We got up early in the morning to enjoy the sunrise scene at the seaside. Kevin had a pitiful blunder. His shoes and trousers were wetted by a sudden strike of waves. That morning the last coordination on Problem 2 was done. Albert was awarded 3 marks for his analytic struggle. The uncertainties of our results then shifted from our actual scores to the medal cutting scores.

We went shopping for souvenirs in the afternoon and as soon as we got back to the hotel, I learned from the Chinese leaders that the cutting scores for Gold, Silver and Bronze Medals were 28, 21 and 14 respectively, all being multiples of 7. I breathed a sigh of relief as my Silver Medal was ultimately secure.

<u>**July 15**</u> In the afternoon we had the Closing Ceremony followed by a chain of photo-taking. We won three Silver Medals (Albert, Jimmy and me), one Bronze Medal (Alice) and two Honorable Mentions (Kit and Kevin).

<u>**July 16, 17 and 18**</u> The six-hour bus journey from Mar del Plata to Buenos Aires passed rapidly in our dreams. Then after a long flight, we were finally home in one piece and me with several bonus pimples.

In conclusion I shall stress one point – succinctly but with all the strength that I command – one can never pay sufficient tribute to our IMO trainers, who have so selflessly devoted countless hours of their own time to Mathematical Olympiad over the years. I can find no words to thank them the way they truly deserve.

"Ask not what your country can do for you; ask what you can do for your country." With this John F. Kennedy exclamation I urge you all to support the 2016 Hong Kong IMO by whatever means you can, so that together we can make it an all-time success.