Mathematical Excalibur

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Olympiad Corner

Below are the problems of the 2017 International Mathematical Olympiad (July 18-19, 2017) held in Brazil.

Problem 1. For each integer $a_0 > 1$, define the sequence $a_0, a_1, a_2, \ldots$ by:

$$a_{n+1} = \begin{cases} \sqrt{a_n} & \text{if } \sqrt{a_n} \text{ is an integer,} \\ a_n + 3 & \text{otherwise,} \end{cases}$$

for each $n \geq 0$. Determine all values of a_0 for which there is a number A such that $a_n = A$ for infinitely many values of n.

Problem 2. Let ℝ be the set of real numbers. Determine all functions f : ℝ→ ℝ such that, for all real numbers x and y, $f(f(x)f(y)) + f(x+y) = f(xy)$.

Problem 3. A hunter and an invisible rabbit play a game in the Euclidean plane. The rabbit's starting point, A_0, and the hunter's starting point, B_0, are the same. After $n-1$ rounds of the game, the rabbit is at point A_{n-1} and the hunter is at point B_{n-1}. In the n^{th} round of the game, three things occur in order.

(continued on page 4)

Editors: 高子眉 (KO Tsz-Mei)
梁達榮 (LEUNG Tat-Wing)
李健賢 (LI Kin-Yin), Dept. of Math., HKUST
吳鏡波 (NG Keng-Po Roger), ITC, HKPU

Artist: 楊秀英 (YEUNG Sau-Ying Camille), MFA, CU

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The editors welcome contributions from all teachers and students. With your submission, please include your name, address, school, email, telephone and fax numbers (if available). Electronic submissions, especially in MS Word, are encouraged. The deadline for receiving material for the next issue is ***October 21, 2017***.

For individual subscription for the next five issues for the 17-18 academic year, send us five stamped self-addressed envelopes. Send all correspondence to:

Dr. Kin-Yin LI, Math Dept., Hong Kong Univ. of Science and Technology, Clear Water Bay, Kowloon, Hong Kong

Fax: (852) 2358 1643
Email: makyli@ust.hk

Notes on IMO2017

Tat Wing Leung

International Mathematical Olympiad (IMO) 2017 was held in Rio De Janeiro, Brazil from 12 to 24 July, 2017. Members of Hong Kong Team are as follows.

Tat Wing Leung (Leader)

Tak Wing Ching (Deputy Leader)

Man Yi Mandy Kwok, Shun Ming Samuel Lee, Yui Hin Arvin Leung, Cheuk Hin Alvin Tse, Jeff York Ye, Hoi Wai Yu (Contestants)

All contestants except Alvin Tse are entering universities during the academic year 2017-18. Thus we will have an essentially new team next year.

I went first to Brazil in July 13. Professor Shum Kar Ping, chairman of our Committee also went with me. He was to present the report of IMO2016. It was over quickly. Apparently members of the Advisory Board had nothing more to ask. Luckily it was done.

Upon arrival, I just had to follow the program closely and to attend Jury meetings. As claimed, I did experience *the famous Brazilian Hospitality* (this clause was copied from the program book) and I was quite happy in general.

As in these few years, in choosing the problems, first 4 easy problems, 1 from each of the four categories (Algebra, Combinatorics, Geometry and Number Theory) were selected. Then 4 medium problems, again 1 from each category was selected. Then members of the Jury (leaders) selected two easy problems of two categories, and the 2 medium problems from the two complementary categories were selected. It was claimed this scheme will help to produce a more balanced paper. But after a few years, I do think it is not necessarily true. First almost certain an easy geometry problem will be selected, thus all medium but nice geometry problems will be discarded. It is also almost certain two combinatorics problems will be selected. The papers will then become more predictable. Anyway members still chose this scheme.

Our contestants arrived on July 16. During the opening ceremony, July 17, I had a chance to look at them (from far away). In the opening ceremony, the speech of Marcelo Viana, director of IMPA (Instituto de Mathematica Pura e Applicada) was particularly genuine and moving. He talked about the IMO training and selection in Brazil in these 38 years. (Certainly it was not an easy task to select a team of 6 from 18 million youngsters). Then he also talked about Maryam Mirzakhani, the Iranian Mathematician, who was a 1994 and 1995 IMO gold medalist, 2014 Fields' medalist and passed away prematurely at age 40. Finally, he also talked about the upcoming International Congress of Mathematicians (ICM) 2018, to be held in Brazil.

The next two days (July 18 and 19) are contest days. The contestants had to sit for two 4.5 hours exam during the mornings. In the first half hours of the exams, there were Q&A times. In this year again they adapted a new scheme, namely they had 4 tables, 3 tables for problems 1, 2 and 3 (problems 4, 5 and 6 the next day), and so they were 4 queues. Clearly this is a more efficient scheme than before.

Again the next two days (July 20 and 21) were days of coordination, namely leaders and coordinators would decide the score of a particular problem. We followed the schedule to go to a particular table. We had a very capable deputy leader this year and so he knew well what our team members had done. So the process became relatively easy.

(continued on page 2)

Since the problems of IMO2017 is listed in this issue of Excalibur, I shall not reproduce them here nor to copy the proofs. I will only give a few comments of this year's problems. First a few key words came to my mind. My first word is algorithm (or construction). Indeed the proposer has been trying hard to think of a new scenario that when you try to solve the problem, you need to invent a new algorithm to solve the problem. For example, problems 5 and 6 do not need to know a lot of higher math, but you do need to have some sense of ingenuity to think of a new scheme or method to solve a particular problem. In problem 3, you had to show an algorithm does not exist. The second word is induction, namely in these problems, small cases (cases with smaller parameters) were easy. So one might try to consider if the method of induction does work. It was not obvious. The third word is geometry. In this year, only 3 of us could solve the relatively easy geometry problem. Indeed this year's geometry problem (P4), no new constructions are required, no new transformations (inversion, homothety, etc.) are needed. It is simply correct drawing and angle chasing. So I must admit that we have reverted back to our usual tradition.

Now I will say a few more words on the individual problems and the performance of our team. Problem 1 is a number theory problem. Once a contestant tries a few cases and guess the correct answer ($a_0 \equiv 0 \pmod 3$), then it is not too hard to prove $a_0 \equiv 1,2 \pmod 3$ do not work but $a_0 \equiv 0 \pmod 3$ works. Our team this year is relatively mature and relatively well trained. So all of them solved the problem and we have a perfect score.

Problem 2 is a functional equation, showing $f(f(x)f(y))+f(x+y)=f(xy)$ for all real x and y will imply $f(x)=0$ or $f(x) = \pm(x-1)$. The most troublesome thing is the marking scheme. It is easy to get the first 3 points, but it is real hard to get an extra point, i.e., proving injectivity and onward. A leader secretly showed me the scores of problem 2 of his team, apparently he was dismayed by the performance. I was not sure. At the end I found their team scored 1 more point than us.

For problem 3, I had (and still have) a very serious concern about it. Observe only two contestants scored 7 points (a Russian and an Australian contestant), and also none of the USA team and the Chinese team (plus other teams) together scored any point at all. I suspect many contestants are like me and simply don't know what exactly is going on. Indeed it is not quite sure what it means by "no matter how" and what exactly it means by a tracking device, I was told it is not like the "best strategy". Indeed when you look at the solution, you get the idea such a strategy (or algorithm) does not exist. The solution is roughly as follows. Assume the rabbit moves in a straight line, and with luck (this term appears quite a few times in the solution) the tracking device also moves in a straight line. Because of this happening, the hunter can only move along a straight line (also with no justification but intuition) and follow the rabbit, and after finitely many steps, the distance between the rabbit and the hunter will only increase (easy to show by simple geometry). Thus there is no best strategy. I am still awaiting members to educate me on this problem.

Problem 4 was a relatively easy geometry exercise.

We did best in problem 5 among all teams, (our deputy leader reminded me about this point). Indeed altogether we scored 26 points. So essentially 4 of us solved the problem, while other teams scored at most 23 points. This shows our team does know something about problem solving. Indeed the problem is equivalent to say there are $N(N+1)$ distinct integers randomly placed in a row, say, you can throw away $N(N-1)$ of them, and among the remaining integers, the largest integer and the second largest integer will stick together, so are the third largest and the fourth largest integer will stick together, and so on. Not too hard?

For Problem 6, an ordered pair (x,y) of integers is a primitive point if $\gcd(x,y)=1$. Now given a set of finitely many primitive points (x_i, y_i), $1 \le i \le n$, we need to find a homogeneous polynomial $g(x,y)$ such that $g(x_i,y_i)=1$. If there is only one primitive point, then it is trivial, by Euclidean algorithm. The hard part is how to move on by induction. But it is not at all easy.

At the end Shun Ming was awarded a gold medal (25 points), Mandy a silver (23 points), Jeff (18 points), Hoi Wai (17 points) and Cheuk Hin (17 points) all received Bronze medals. Yui Hin (11 points) managed to get a honorable mention. Our rank is 26 among 111 countries/regions. Surely the result was not as good as last year nor as we had hoped for. Nevertheless there were certain things we can say. Indeed this was the 30th consecutive year that we sent teams to IMOs. No matter what, it is not an easy matter and it should be a date to remember. (Better still, we hosted the event in 1994 and 2016). Also Mandy Kwok was the second girls among all girl contestants. IMPA this year gave out 5 prizes to female contestants. Initially I thought Mandy should have a chance to get a prize. Later I found out the prizes were for the top female students who contribute the most to their respective team's score. So I understand why she was not eligible for the prize. Nevertheless I must say we are very glad to see her improving very well in these few years. Finally we managed to get the highest score in Problem 5. I think this is an indication that our team is comparable with any other team. They really don't have much special recipe we don't envisage.

I hasten to say the cut scores of IMO this year cannot be said to be ideal. Indeed the cut scores for gold is 25, for silver 19, and bronze 16. One may say the easy problems (problems 1 and 4) were too easy and the four other problems too hard. The easy problem were too easy. Hence 14 points was not enough for a bronze and the hard problems too hard. Thus 25 points was good enough to get a gold. Really we expect a contestant to solve at least 2 problems (≥ 14 points) to get a bronze, at least 3 problems (≥ 21 points) a silver, and at least 4 problems (≥ 28 points) to get a gold. Some people expect a contestant should solve nearly at least 5 problems to get a gold. Really what is the point to set a problem so that only 2 out of 615 contestants can solve it?

Since we are trailing behind some other Asian countries this year, it was suggested that more money should be put into this activity. In my opinion the stakeholders (members of the Committee, the Academy and the Gifted Section of Education, but most important of all, past and present trainees) should sit together and sort out what exactly do we want, how much money/resource should be put into it and who will contribute what, etc. I suppose it is time to start thinking.

Problem Corner

We welcome readers to submit their solutions to the problems posed below for publication consideration. The solutions should be preceded by the solver's name, home (or email) address and school affiliation. Please send submissions to Dr. *Kin Y. Li, Department of Mathematics, The Hong Kong University of Science & Technology, Clear Water Bay, Kowloon, Hong Kong.* The deadline for sending solutions is ***October 21, 2017.***

Problem 501. Let x, y, s, m, n be positive integers such that $x+y=s^m$ and $x^2+y^2=s^n$. Determine the number of digits s^{300} has in base 10.

Problem 502. Let O be the center of the circumcircle of acute $\triangle ABC$. Let P be a point on arc BC so that A, P are on opposite sides of side BC. Point K is on chord AP such that BK bisects $\angle ABC$ and $\angle AKB > 90°$. The circle Ω passing through C, K, P intersect side AC at D. Line BD meets Ω at E and line PE meets side AB at F. Prove that $\angle ABC = 2\angle FCB$.

Problem 503. Let S be a subset of $\{1,2,\ldots,2015\}$ with 68 elements. Prove that S has three pairwise disjoint subsets A, B, C such that they have the same number of elements and the sums of the elements in A, B, C are the same.

Problem 504. Let $p>3$ be a prime number. Prove that there are infinitely many positive integers n such that the sum of k^n for $k=1,2,\ldots,p-1$ is divisible by p^3.

Problem 505. Determine (with proof) the least positive real number r such that if z_1, z_2, z_3 are complex numbers having absolute values less than 1 and sum 0, then

$$|z_1z_2+z_2z_3+z_3z_1|^2 + |z_1z_2z_3|^2 < r.$$

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Solutions

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Problem 496. Let a,b,c,d be real numbers such that $a+\sin b > c+\sin d$, $b+\sin a > d+\sin c$. Prove that $a+b>c+d$.

Solution. **Toshihiro SHIMIZU** (Kawasaki, Japan).

For $x\ge 0$, $|\sin x|\le x$. Let $s = a - c$ and $t = d - b$. We have

$$\begin{aligned} s = a-c &> \sin d - \sin b \\ &= 2\cos[(d+b)/2]\sin[(d-b)/2] \\ &\ge -2|\sin(t/2)| \end{aligned}$$

and

$$\begin{aligned} t = d-b &< \sin a - \sin c \\ &= 2\cos[(a+c)/2]\sin[(a-c)/2] \\ &\le 2\,|\sin(s/2)|. \end{aligned}$$

If $s\ge 0$, then $t< 2|\sin(s/2)|\le s$. Similarly, if $t\le 0$, then $s > -2|\sin(-t/2)| \ge -2(-t/2) = t$.

Finally, if $s < 0 < t$, then $-s < 2|\sin(t/2)| \le t$ and $t < 2|\sin(s/2)| = |\sin(-s/2)| \le -s$, which leads to a contradiction.

Comment: The above solution avoided calculus as it used $\sin x \le x$ for $0\le x\le 1$, which followed by taking points A, B on a unit circle with center O such that $\angle AOB = 2x$, then the length $2x$ of arc AB is greater than the length $2\sin x$ of chord AB.

Other commended solvers: **Jason FONG** and **LW Solving Team** (S.K.H. Lam Woo Memorial Secondary School).

Problem 497. Let there be three line segments with lengths 1, 2, 3. Let the segment of length 3 be cut into $n\ge 2$ line segments. Prove that among these $n+2$ segments, there exist three of them that can be put to form a triangle where each side is one of the three segments.

Solution. **William FUNG**, **Mark LAU** (Pui Ching Middle School), **LW Solving Team** (S.K.H. Lam Woo Memorial Secondary School) and **Toshihiro SHIMIZU** (Kawasaki, Japan).

Note line segments with lengths $a\le b\le c$ form a triangle if and only if $a+b>c$. Let $a_1\le a_2\le\cdots\le a_n$ be the lengths of such n segments with sum equals to 3. Assume there exists i such that $a_i>1$. If $1<a_i<2$, then the segments with length $1,a_i,2$ forms a triangle since $1+a_i>2$. If $2\le a_i$, then the segments with length $1,2,a_i$ forms a triangle since $1+2>a_i$. It remains to consider the case all $a_i\le 1$. Then $i\ge 3$.

Assume no 3 of these segments form a triangle. Then $a_1+a_2\le a_3$, $a_2+a_3\le a_4$, …, $a_{n-2}+a_{n-1}\le a_n$, $a_n+1\le 2$. Adding these and cancelling $a_3,\ldots,a_n,1$ on both sides, we have

$$3+a_2 = (a_1+a_2+\cdots+a_n)+a_2 \le 2,$$

which yields $a_2\le -1$, a contradiction.

Problem 498. Determine all integers $n>2$ with the property that there exists one of the numbers $1,2,\ldots,n+1$ such that after its removal, the n numbers left can be arranged as $a_1,a_2,\ldots,a_n$ with no two of $|a_1-a_2|$, $|a_2-a_3|$, …, $|a_{n-1}-a_n|$, $|a_n-a_1|$ being equal.

Solution. **LW Solving Team** (S.K.H. Lam Woo Memorial Secondary School), **George SHEN** and **Toshihiro SHIMIZU** (Kawasaki, Japan).

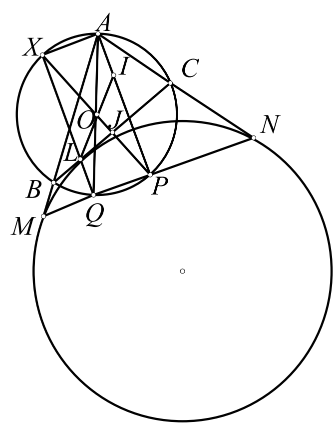
Since no two of $|a_1-a_2|$, $|a_2-a_3|$, …, $|a_{n-1}-a_n|$, $|a_n-a_1|$ being equal and each is at most n, they must be $1,2,\ldots,n$ in some order. So $|a_1-a_2| + |a_2-a_3| + \cdots + |a_{n-1}-a_n|+|a_n-a_1|=1+2+\cdots+n=n(n+1)/2$. From $a \equiv |a| \pmod 2$ and $(a_1-a_2)+(a_2-a_3)+\cdots+(a_{n-1}-a_n)+(a_n-a_1)=0$, we see $|a_1-a_2|+ |a_2-a_3| + \cdots + |a_{n-1}-a_n| + |a_n-a_1|$ is even. For $n(n+1)/2$ to be even, this implies $n \equiv 0$ or $-1 \pmod 4$.

In the case $n=4k$, remove $k+1$ and let $a_1=4k+1$, $a_2=1$, $a_3=4k$, $a_4=2$, …, $a_{2k-1}=3k+2$, $a_{2k}=k$, $a_{2k+1}=3k+1$, $a_{2k+2}=k+2$, $a_{2k+3}=3k$, $a_{2k+4}= k+3$, …, $a_{4k-1}=2k+2$ and $a_{4k}=2k+1$.

In the case $n=4k-1$, remove $3k$ and let $a_1=4k$, $a_2=1$, $a_3=4k-1$, $a_4=2$, …, $a_{2k-1}=3k+1$, $a_{2k}=k$, $a_{2k+1}=3k-1$, $a_{2k+2}=k+1$, $a_{2k+3}=3k-2$, …, $a_{4k-2}= 2k-1$, $a_{4k-1}=2k$.

Problem 499. Let ABC be a triangle with circumcenter O and incenter I. Let Γ be the escribed circle of $\triangle ABC$ meeting side BC at L. Let line AB meet Γ at M and line AC meet Γ at N. If the midpoint of line segment MN lies on the circumcircle of $\triangle ABC$, then prove that points O, I, L are collinear.

***Solution*. George SHEN.**



Let P be the midpoint of MN. From $AM=AN$, we see $AP\perp MN$. So A,I,P are collinear. Let Q be on MN such that $LQ\perp MN$. Now $\angle BMQ=\angle CNQ$ and

$$\frac{MQ}{NQ} = \frac{ML\cos\angle LMQ}{NL\cos\angle LNQ} = \frac{2MB\cos\angle LMB\cos\angle LNC}{2NC\cos\angle LNC\cos\angle LMB} = \frac{MB}{NC}.$$

This implies $\triangle BMQ$, $\triangle CNQ$ are similar.

Let $a=BC$, $b=CA$, $c=AB$, $s=(a+b+c)/2 = AM = AN$ and $\alpha=\angle BAC$.

We have

$$AP = AM\cos(\alpha/2) = s\cos(\alpha/2).$$

By extended sine law, $BC = a = 2R\sin\alpha$. From $IP=BP=CP$ [see Math Excalibur, *vol.* 11, *no.* 2, *page* 1, *Theorem in middle column–Ed.*], we have

$$a = BC = 2BP\sin\frac{180^\circ-\alpha}{2} = 2BP\cos\frac{\alpha}{2},$$

$$\cos\frac{\alpha}{2} = \frac{a}{2IP} = \frac{a}{2(AP-AI)} = \frac{a}{2(s\cos\frac{\alpha}{2}-AI)}.$$

Applying $AI\cos(\alpha/2)=s-a$ and the last equation, we can get

$$2s\cos^2\frac{\alpha}{2} = 2s-a = b+c,$$

$$2s\sin^2\frac{\alpha}{2} = a.$$

Next $MN = 2AM\sin(\alpha/2) = 2s\sin(\alpha/2)$ and $(MQ+NQ)\sin(\alpha/2) = MB+NC$. Using $MQ/NQ=MB/NC$, we get

$$MQ\sin(\alpha/2)=MB$$

and

$$NQ\sin(\alpha/2)=NC,$$

which says $\angle QBA = 90^\circ = \angle QCA$. Then Q is on Γ and AQ is a diameter of Γ.

Let line LQ meet the circumcircle Γ of $\triangle ABC$ at X as labeled in the figure. Observe that $APQX$ is a rectangle and AQ, XP are diameters of Γ intersecting at O. We claim $LQ=AI$ (then $LI\cap AQ$ at O and so O,I,L are collinear).

Now $BO=CO$, $BJ=CJ$ and $\angle BAP = \angle CAP$ implies $BP=CP$. Hence, O, J, P are collinear. Next $OJ\perp BC$ implies $\angle LJP=90^\circ=\angle LQP$. Then, J,P,Q,L are concyclic. Hence,

$$XL\cdot XQ = XJ\cdot XP$$

Let R be the circumradius of $\triangle ABC$. From

$$XJ = \frac{a}{2}\cot\frac{\alpha}{2},\ XP = 2R,$$

$$IP = 2R\sin\frac{\alpha}{2},\ AP = s\cos\frac{\alpha}{2},$$

We get $XJ\cdot XP = IP\cdot AP$. Then $XL\cdot XQ = IP\cdot AP$. Since $XQ=AP$, so $XL=IP$. Then $QL=XQ-XL=AP-IP=AI$. The conclusion follows.

Other commended solvers: **LW Solving Team** (S.K.H. Lam Woo Memorial Secondary School) and **Toshihiro SHIMIZU** (Kawasaki, Japan).

Problem 500. Determine all positive integers n such that there exist $k\geq 2$ positive rational numbers such that the sum and the product of these k numbers are both equal to n.

Solution. **Mark LAU** (Pui Ching Middle School), **LW Solving Team** (S.K.H. Lam Woo Memorial Secondary School) and **Toshihiro SHIMIZU** (Kawasaki, Japan).

Observe that for a composite number n, there exist integer $s,t\geq 2$ such that $n=st$, the sequence $s,t,1,1,\ldots,1$ (with $st-s-t$ 1's) has sum and product equals $st=n$.

For prime numbers $n\geq 11$, the sequence $n/2,1/2,2,2,1,1,\ldots,1$ (with $n-4-(n+1)/2$ 1's) satisfies the condition by a simple checking.

For $n=7$, the sequence 9/2, 4/3, 7/6, satisfies the condition by a simple checking.

Next we claim the cases $n=1,2,3,5$ have no solution. Assume $a_1, a_2,\ldots,a_k$ are positive rational numbers with sum and product equals to n. By the AM-GM inequality, we have

$$\frac{n}{k} = \frac{a_1+\cdots+a_k}{k} \geq \sqrt[k]{a_1\cdots a_k} = \sqrt[k]{n}.$$

Then $n\geq k^{k/(k-1)}>k$. Since $n>k\geq 2$, cases $n=1$ or 2 are impossible.

Finally, for $n=3$ or 5, since $3^{3/(3-1)}=5.1\ldots$ implies $k=2$, so only cases (n,k) = (3,2) and (5,2) remain. Now

$$\begin{aligned}(a_1-a_2)^2 &= (a_1+a_2)^2-4a_1a_2\\ &= n^2-4n = -3 \text{ or } 5,\end{aligned}$$

which have no rational solutions a_1, a_2. Therefore, the answers are all positive integers except 1,2,3,5.

Olympiad Corner

(*Continued from page 1*)

Problem 3 (Cont'd).

(i) The rabbit moves invisibly to a point A_n such that the distance between A_{n-1} and A_n is exactly 1.

(ii) A tracking device reports a point P_n to the hunter. The only guarantee provided by the tracking device to the hunter is that the distance between P_n and A_n is at most 1.

(iii) The hunter moves visibly to a point B_n such that the distance between B_{n-1} and B_n is exactly 1.

Is it always possible, no matter how the rabbit moves, and no matter what points are reported by the tracking device, for the hunter to choose her moves so that after 10^9 rounds she can ensure that the distance between her and the rabbit is at most 100?

Problem 4. Let R and S be different points on a circle Ω such that RS is not a diameter. Let ℓ be the tangent line to Ω at R. Point T is such that S is the midpoint of the line segment RT. Point J is chosen on the shorter arc RS of Ω so that the circumcircle Γ of triangle JST intersects ℓ at two distinct points. Let A be the common point of Γ and ℓ that is closer to R. Line AJ meets Ω again at K. Prove that the line KT is tangent to Γ.

Problem 5. An integer $N\geq 2$ is given. A collection of $N(N+1)$ soccer players, no two of whom are of the same height, stand in a row. Sir Alex wants to remove $N(N-1)$ players from this row leaving a new row of $2N$ players in which the following N conditions hold:

(1) no one stands between the two tallest players,

(2) no one stands between the third and fourth tallest players,

⋮

(N) no one stands between the two shortest players.

Show that this is always possible.

Problem 6. An ordered pair (x,y) of integers is a primitive point if the greatest common divisor of x and y is 1. Given a finite set S of primitive points, prove that there exist a positive integer n and integers $a_0, a_1, \ldots, a_n$ such that, for each (x,y) in S, we have:

$$a_0x^n + a_1x^{n-1}y + a_2x^{n-2}y^2 + \cdots + a_{n-1}xy^{n-1} + a_ny^n = 1.$$