Mathematical Excalibur

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Olympiad Corner

Below were the Hong Kong (China) Mathematical Olympiad on December 1, 2018.

Problem 1. Given that *a, b* and *c* are positive real numbers such that $ab+bc+ca\geq 1$, prove that

$$\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\geq\frac{\sqrt{3}}{abc}.$$

Problem 2. Find the number of nonnegative integers k, $0\leq k\leq 2188$, and such that $2188!/(k!(2188-k)!)$ is divisible by 2188.

Problem 3. The incircle of ΔABC, with incenter I, meets BC, CA and AB at D, E, F respectively. The line EF cuts the lines BI, CI, BC and DI at points K, L, M and Q respectively. The line through the midpoint of CL and M meets CK at P.

(a) Determine $\angle BKC$.
(b) Show that the lines PQ and CL are parallel.

Problem 4. Find all integers $n\geq 3$ with the following property: there exist n distinct points on the plane such that each point is the circumcenter of a triangle formed by 3 of the points.

Acknowledgment: Thanks to Sindy Ting, Math. Dept., HKUST for general assistance.

On-line: http://www.math.ust.hk/excalibur/

The editors welcome contributions from all teachers and students. With your submission, please include your name, address, school, email, telephone and fax numbers (if available). Electronic submissions, especially in MS Word, are encouraged. The deadline for receiving material for the next issue is ***November 2, 2019***.

For individual subscription for the next five issues for the 18-19 academic year, send us five stamped self-addressed envelopes. Send all correspondence to:

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Notes on IMO 2019

Tat Wing LEUNG

Despite all its sham, drudgery and broken dreams, the Gifted Section of the Education Department (EDB), the Hong Kong Academy of Gifted Education (HKAGE), and our Committee (International Mathematical Olympiad Hong Kong Committee, IMOHKC) managed to send a team to the 60th International Mathematical Olympiad (IMO 2019). The competition was held from July 11 to July 22, 2019, in Bath, United Kingdom.

The team was composed as follows: Leader: Leung Tat Wing, Deputy Leader: Cesar Jose C. Jr. Alaban (CJ), Members: Bruce Changlong Xu, Daniel Weili Sheremeta, Harris Leung, Wan Lee, Nok To Omega Tong, Sui Kei Ho. A lady from EDB (Miriam Cheung) also went with us as an observer.

Let me briefly discuss the problems of the two contests.

Problem 1 was very interesting. It was initially selected as the easy algebra problem and later selected as the easy pair. Although it was most liked, it was also most hated. I supposed it was because some leaders thought the problem was simply too easy. By substituting suitable values (say a by 0 and b by $n+1$ one quickly comes to the conclusion that the function is linear (or by Cauchy), and hence by using some initial values to get the answers. Some leaders first tried to replace the easy algebra by another easy problem (which was actually classified as a combinatorial problem), and later tried to add alternate option pairs to the option pairs that contained the easy algebra problem. I myself could not say if it was right or wrong, I just found it funny. Indeed the problem was selected using the approach as agreed, why tried to change it in the middle of the process? At the end of the day, totally 73 students did not get anything in this problem, and only slightly more than half (382 out of 621) scored full mark.

Problem 4 was an easy Diophantine equation. By putting small values of n, one quickly comes up with the solutions (1,1) and (3,2), the hard part is to show that there are no more. Many students lost partial marks while trying to compare values (or 2-adic valuations) of the two sides of the equation. As learned from leaders of stronger teams, I found they considered Legendre's formula and/or the lifting exponent lemma rather common tools, although the lemma was not really necessary. So yes, do we need to ask our students to further enhance their toolkit?

Problem 5 was an *ouroboros*-type problem, namely part of the problem is relating to other part of itself. In this case we are given a sequence of heads and tails of n coins, the k^{th} coin is flipped if there are exactly k heads in the sequence. The problem is not too hard, and given its "natural" condition, it is probably known. Indeed if the first coin is head, then basically we need to deal with the remaining sequence of length $n-1$, and the final step is to flip the first coin. If the last coin is a tail, then it will never be flipped, and we are basically dealing with the first $n-1$ coins.

If the first coin is a tail, and the last coin is a head, then we first deal with the middle $n-2$ coins. After that only one head remaining (at the end), then the first $n-1$ coins are flipped successively and all become heads, then starting from the end, each coin is flipped, until the first one and every coin becomes tail. Using these, we can make up recursive relations and get the answer relatively easy. Our team members, using their own ingenuity and persistence, managed to do the problem well.

(continued on page 4)

Wilson's Theorem

Kin Y. Li

In solving number theory problems, Fermat's or Euler's theorems as well as the Chinese remainder theorem are often applied. In this article, we will look at examples of number theory problems involving factorials. For this type of problems, Wilson's theorem asserts that for every prime number p, we have $(p-1)! \equiv -1 \pmod p$. Below are problems using Wilson's theorem.

Problem 1. Let p be an odd integer greater than 1. Prove that

$$1^2\cdot 3^2\cdot 5^2\cdots(p-2)^2 \equiv (-1)^{(p+1)/2} \pmod p.$$

Solution. By Wilson's theorem, $(p-1)! \equiv -1 \pmod p$ when p is an odd prime. Also, we have $i \equiv -(p-i) \pmod p$. Multiplying the cases $i = 1, 3, \cdots, p-2$, we get

$$1\cdot 3\cdots(p-2) \equiv (-1)^{(p-1)/2}(p-1)(p-3)\cdots 2 \pmod p.$$

Multiplying both sides by $1\cdot 3\cdots(p-2)$, we get

$$1^2\cdot 3^2\cdot 5^2\cdots(p-2)^2 \equiv (-1)^{(p-1)/2}(p-1)! \equiv (-1)^{(p+1)/2} \pmod p.$$

Problem 2. Let p be a prime number and $N = 1+2+3+\cdots+(p-1) = (p-1)p/2$. Prove that $(p-1)! \equiv p-1 \pmod N$.

Solution. Since p is prime, by Wilson's theorem, $(p-1)! \equiv -1 \pmod p$. Then there exists an integer m such that

(*) $(p-1)! = mp-1 = (m-1)p+(p-1)$.

So $(m-1)p = (p-1)!-(p-1) = (p-1)k$, where $k=(p-2)!-1$ and $p|(p-1)k$. Since $\gcd(p,p-1)=1$, so $p|k$. Let $k=np$, then

(**) $(m-1)p=(p-1)pn$,

so $m-1=n(p-1)$. Putting (**) into (*), we get

$$(p-1)! = [n(p-1)+1]\,p-1 = n(p-1)p+p-1 = 2n[(p-1)p/2]+p-1 = 2nN+p-1.$$

So $(p-1)! \equiv p-1 \pmod N$.

Problem 3. Determine all positive integers n having the property that there exists a permutation $a_1, a_2, \ldots, a_n$ of $0,1,2,\ldots,n-1$ such that when divided by n, the remainders of $a_1, a_1a_2, \ldots, a_1a_2\cdots a_n$ are distinct.

Solution. When n is a prime number p, let $a_1=1$ and other integers a_i satisfy $0\le a_i\le p-1$ and $ia_{i+1} \equiv i+1 \pmod p$ for $i = 2,\cdots, p$.

Then $a_1, a_1a_2, \ldots, a_1a_2\cdots a_n$ when divided by n have remainders $1,2,\cdots, p$. Also, from $ia_{i+1} \equiv i+1 \pmod p$, we see $a_{i+1}-1$ is the inverse of i. So $a_1, a_2, \ldots, a_n$ are distinct.

When $n = 1$ or 4, the permutations (0), $(1,3,2,0)$ satisfy the condition. When $n>4$ is composite, if $n = p^2$, let $q = 2p < n$. Otherwise $n=pq$ with $1<p<q<n$ so that $pq|(n-1)!$.

If the required permutation exists, then $a_n=0$ and $a_1a_2\cdots a_{n-1}=(n-1)!\equiv 0 \pmod n$, which is a contradiction. (In fact, when $n>4$ is composite, $n \mid (n-1)!$ and $3! \equiv -2 \pmod 4$ so that the converse of Wilson's theorem also hold.

Problem 4. For integers n, q satisfying $n\ge 5$ and $n\ge q\ge 2$, prove that $[(n-1)!/q]$ is divisible by $q-1$.

Solution. (1) If $n>q$, then $(q-1)q \mid (n-1)!$. Hence, $(q-1) \mid [(n-1)!/q]$.

(2) If $q=n$ and q is composite, then $[(n-1)!/q]=(n-1)!/n$. Since $\gcd(n-1,n)=1$ and $q-1=(n-1) \mid (n-1)!$. So $q-1$ divides $[(n-1)!/q]$.

(3) If $q=n$ is prime, then by Wilson's theorem, $(n-1)! \equiv -1 \pmod n$ so that $(n-1)!+1=kn$ for some integer k. Then $[(n-1)!/q]=k-1$ and $(k-1)n=(n-1)!+1-n$ so that $k-1=((n-2)!-1)(n-1)/n$ is an integer. Since $\gcd(n-1,n)=1$, so n divides $(n-2)!-1$. Therefore, $[(n-1)!/q]=k-1$ is a multiple of $n-1$.

Problem 5. Let $P(x)=a_nx^n+a_{n-1}x^{n-1}+\cdots+a_1x+a_0$, where $a_0, a_1, \cdots, a_n$ are integers, $a_n>0$ and $n\ge 2$. Then prove that there exists a positive integer m such that $P(m!)$ is a composite number.

Solution. If $a_0=0$, then $m! \mid P(m!)$ and the conclusion follows.

Next let $S(x) = a_0x^n + a_1x^{n-1} + \cdots + a_n$. Suppose $a_0\ne 0$. By Wilson's theorem, for every prime p and positive even integer $k < p$, we have

$$\begin{aligned}&(k-1)!(p-k)!\\ &\equiv (-1)^{k-1}(p-k)!(p-k+1)(p-k+2)(p-1)\\ &= -(p-1)! \equiv 1 \pmod p.\end{aligned}$$

So $(p-1)! \equiv -1 \pmod p$ and

$$((k-1)!)^n\, P((p-k)!) \equiv S((k-1)!) \pmod p.$$

So $p \mid P((p-k)!)$ if and only if $p \mid S((k-1)!)$. Take $k > 2a_n+1$. Then $u = (k-1)!/a_n$ is an integer divisible by all primes not greater than k.

Problem 6. If p and $p+2$ are both prime numbers, then we say they are twin primes. Show that if p and $p+2$ are twin primes, then $4(p-1)!+4+p$ is divisible by $p(p+2)$.

Solution. If p and $p+2$ are prime, then $p>2$ so that p and $p+2$ are odd. By Wilson's theorem, $(p-1)! \equiv -1 \pmod p$ and also $(p+1)! \equiv -1 \pmod{p+2}$. Then we have

$$4(p-1)!+4+p \equiv 0 \pmod p.$$

Also

$$\begin{aligned}4(p-1)!+4 &\equiv -p(p+1)p[(p-1)!+1]\\ &\equiv -p[(p+1)!+2] \equiv -p \pmod{p+2},\end{aligned}$$

which is $4(p-1)!+4+p \equiv 0 \pmod{p+2}$. As $\gcd(p,p+2)=1$, we get $4(p-1)!+4+p \equiv 0 \pmod{p(p+2)}$.

Problem 6. (Wolstenholme's Theorem) Let p be a prime greater than or equal to 5. For positive integers m and n that are relatively prime and

$$\frac{m}{n} = \frac{1}{1^2} + \frac{1}{2^2} + \cdots + \frac{1}{(p-1)^2}.$$

Prove that p is a divisor of m and p^2 is a divisor of

$$(p-1)!\left(1+\frac{1}{2}+\cdots+\frac{1}{p-1}\right).$$

Solution. If integer k is not divisible by p, then there are integers a, b such that $ak+bp = \gcd(k,p) = 1$. We say a is the inverse of k in mod p and denote a as k^{-1}. We have

$$\begin{aligned}((p-1)!)^2\frac{m}{n} &= \sum_{k=1}^{p-1}\frac{((p-1)!)^2}{k^2}\\ &\equiv (-1)^2(1^2+2^2+\cdots+(p-1)^2)\\ &\equiv \frac{(p-1)p(2p-3)}{6} \equiv 0 \pmod p.\end{aligned}$$

Since $\gcd((p-1)!, p) = 1$, so $p \mid m$. Next, let $S=(p-1)!(1+1/2+\cdots+1/(p-1))$. Then

$$\begin{aligned}2S &= (p-1)!\sum_{i=1}^{p-1}\left(\frac{1}{i}+\frac{1}{p-i}\right)\\ &= p\sum_{i=1}^{p-1}\frac{(p-1)!}{i(p-i)} = pT,\end{aligned}$$

where $2S$, p and T are integers. Since $\gcd(p,2)=1$, so p divides S. Due to $p|m$,

$$T = \sum_{i=1}^{p-1}\frac{(p-1)!}{i(p-i)} \equiv (p-1)!\frac{m}{n} \equiv 0 \pmod p.$$

Problem Corner

We welcome readers to submit their solutions to the problems posed below for publication consideration. The solutions should be preceded by the solver's name, home (or email) address and school affiliation. Please send submissions to *Dr. Kin Y. Li, Department of Mathematics, The Hong Kong University of Science & Technology, Clear Water Bay, Kowloon, Hong Kong*. The deadline for sending solutions is ***November 2, 2019.***

Problem 536. Determine whether there exists a function $f:\mathbb{R}\to\mathbb{R}$ such that for all real x, we have $f(x^3+x)\le x \le (f(x))^3+f(x)$.

Problem 537. Distinct points A, B, C are on the unit circle Γ with center O inside ΔABC. Suppose the feet of the perpendiculars from O to sides BC, CA, AB are D, E, F. Determine the largest value of $OD+OE+OF$.

Problem 538. Determine all prime numbers p such that there exist integers a and b satisfying $p=a^2+b^2$ and a^3+b^3-4 is divisible by p.

Problem 539. In an exam, there are 5 multiple choice problems, each with 4 distinct choices. For every problem, every one of the 2000 students is required to choose exactly 1 of the 4 choices. Among the 2000 exam papers received, it is discovered that there exists a positive integer n such that among any n exam papers, there exist 4 such that for every 2 of the exam papers, there are at most 3 problems having the same choices. Determine the least such n.

Problem 540. Do there exist a positive integer k and a non-constant sequence $a_1, a_2, a_3, \ldots$ of positive integers such that $a_n=\gcd(a_{n+k}, a_{n+k+1})$ for all positive integer n?

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Solutions

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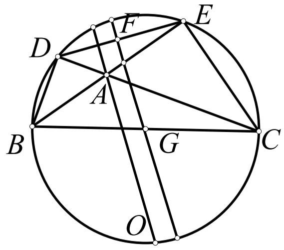
Problem 531. $BCED$ is a convex quadrilateral such that $\angle BDC = \angle CEB = 90°$ and BE intersects CD at A. Let F, G be the midpoints of sides DE, BC respectively. Let O be the circumcenter of ΔBAC. Prove that lines AO and FG are parallel.

Solution 1. **Jon GLIMMS, Hei Chun LEUNG** and **Toshihiro SHIMIZU** (Kawasaki, Japan).



Since $\angle CAO = (180° - \angle COA)/2 = 90° - \angle COA/2 = 90° - \angle CBA = 90° - \angle CBE = 90° - \angle CDE = 90° - \angle ADE$, we have OA and DE are perpendicular. Also, since FG passes through the center G of the circle $CEDG$ and midpoint F of chord DE, FG is perpendicular to DE. Thus, both AO, FG are perpendicular to DE. So lines AO and FG are parallel.

Solution 2. **Prithwijit DE** (HBCSE, Mumbai, India).

Let R be the radius of the circumcircle of triangle BAC. As $\angle BAC>90°$, BC is not the diameter of the circle ABC and therefore D and E are outside the circle ABC. Observe that $EA\cdot EB=EO^2-R^2$ and $DA\cdot DC=DO^2-R^2$. Thus

$$\begin{aligned}EO^2-DO^2&=EA\cdot EB-DA\cdot DC\\&=EA^2-DA^2+EA\cdot AB-DA\cdot DC\\&=EA^2-DA^2.\end{aligned}$$

This implies $OA\perp DE$. Now $FG\perp DE$ because G is the centre of the circle passing through B, C, E and D, and F is the midpoint of chord DE of this circle. Therefore, lines AO and FG are parallel.

Other commended solvers: **CHUI Tsz Fung, Andrea FANCHINI** (Cantù, Italy), **Panagiotis N. KOUMANTOS** (Athens, Greece), **LAU Chung Man** (Lee Kau Yan Memorial School), **LW Maths Solving Team** (SKH Lam Woo Memorial Secondary School), **Jim MAN**, **Corneliu MĂNESCU-AVRAM** (Ploieşti, Romania) and **Apostolis MANOLOUDIS**.

Problem 532. Prove that there does not exist a function $f:(0,+\infty)\to(0,+\infty)$ such that for all $x,y>0$,

$$f^2(x)\ge f(x+y)(f(x)+y).$$

Solution. **Jon GLIMMS, Alvin LUKE** (Portland, Oregon, USA) and **Toshihiro SHIMIZU** (Kawasaki, Japan),

Assume such function exists. We have

$$-y\,f(x+y)/f(y)\ge f(x+y)-f(x).$$

Since the left hand side is negative, f must be strictly monotone decreasing. Also, for any positive integer n and positive real number a, taking the sum for $x=a+i/n$, $y=1/n$, where $1\le i\le n-1$, we get

$$-\frac{1}{n}\sum_{i=1}^{n}\frac{f(a+\frac{i+1}{n})}{f(a+\frac{i}{n})}\ge f(a+1)-f(a).$$

By the AM-GM inequality, we have

$$\frac{1}{n}\sum_{i=1}^{n}\frac{f(a+\frac{i+1}{n})}{f(a+\frac{i}{n})}\ge\sqrt[n]{\prod_{i=1}^{n}\frac{f(a+\frac{i+1}{n})}{f(a+\frac{i}{n})}}\ge\sqrt[n]{\frac{f(a+1)}{f(a)}}.$$

Since $f(a+1)/f(a)>1$, the right hand side will converge to 1 when $n\to\infty$. Thus, $f(a+1)-f(a)\le -1$ for all $a>0$. Then, from $f(1)\ge f(2)+1\ge f(3)+2\ge\cdots$, we have $f(1)\ge f(n+1)+n$ for all positive integer n. This shows that $f(1)$ cannot be finite, a contradiction.

Other commended solvers: **Corneliu MĂNESCU-AVRAM** (Ploieşti, Romania), **Apostolos MANOLOUDIS**, **George SHEN** and **Thomas WOO.**

Problem 533. Let $\mathbb{Z}$ and $\mathbb{N}$ be the sets of all integers and all positive integers respectively. Let $r, s\in\mathbb{N}$. Prove that there are exactly $(r+1)^{s+1}-r^{s+1}$ functions $g:[1,s]\cap\mathbb{N}\to[-r,r]\cap\mathbb{Z}$ such that for all $x,y\in[1,s]\cap\mathbb{N}$, we have $|g(x)-g(y)|\le r$.

Solution. **LAU Chung Man** (Lee Kau Yan Memorial School), **George SHEN** and **Thomas WOO.**

If integer k is in $[-r,r]\cap\mathbb{Z}$, then there are $(\min\{r+1,r-k+1\})^s$ functions satisfying the given conditions which attain values only in $\{k,\ldots,k+r\}$. Of these, $(\min\{r,r-k\})^s$ functions attain values only in $\{k+1, \ldots, k+r\}$. Hence, exactly

$$(\min\{r+1,r+1-k\})^s-(\min\{r,r-k\})^s$$

functions satisfying the given conditions have minimum value k.

This expression equals $(r+1)^s-r^s$ for each of the $r+1$ values $k\le 0$, and it equals $(r+1-k)^s-(r-k)^s$ when $k>0$. Thus, the sum of the expression over all $k\le 0$ is $(r+1)((r+1)^s-r^s)$, while the sumof the expression over all $k>0$ is the telescoping sum

$$\sum_{k=1}^{r}((r+1-k)^s-(r-k)^s)=r^s.$$

Adding these two sums, we find that the total number of functions satisfying the given conditions is $(r+1)^{s+1}-r^{s+1}$.

Other commended solvers: **Jon GLIMMS, Michael HUI** and **Jeffrey HUI**, **Hei Chun LEUNG, Alvin LUKE** (Portland, Oregon, USA) and **Toshihiro SHIMIZU** (Kawasaki, Japan).

Problem 534. Prove that for any two positive integers m and n, there exists a positive integer k such that 2^k-m has at least n distinct prime divisors.

Solution. **Toshihiro SHIMIZU** (Kawasaki, Japan).

We show by induction that there is $k\in\mathbb{N}$ such that 2^k-m has at least n odd prime divisors. If m is even, we can write $n=2^es$ (with odd integer s) and take $k>e$ so we have $2^k-m=2^e(2^{k-e}-s)$. Then it is sufficient to show for $m=s$ (odd). Thus, we assume m is odd.

Taking $k\in\mathbb{N}$ such that $2^k-m>1$, we can take an odd prime divisor p of 2^k-m (which is odd). Assume we have $k\in\mathbb{N}$ such that 2^k-m has n odd prime divisors $p_1, p_2, \ldots, p_n$. For any i the pattern of 2^j (mod p_i) is periodic for j, which implies there are $e_i, f_i\in\mathbb{N}$ such that $2^j\equiv m$ (mod p_i) if and only if $j=e_it+f_i$ for some $t\in\mathbb{N}$. Since $p_i>2$, each e_i is greater than 1. Thus, we can take f_i' such that $f_i\not\equiv f_i'$ (mod e_i). By the Chinese remainder theorem, we can take f_i' such that $k'\equiv f_i'$ (mod e_i) and we have $p_i\nmid 2^{k'}-m$ for $1\le i\le n$. We can also select k' such that $2^{k'}-m>1$. Then we can take odd prime divisor p_{n+1} of $2^{k'}-m$, where p_{n+1} is different from any one of $p_1, p_2, \ldots, p_n$. Then we can choose j such that $2^j\equiv m$ (mod p_{n+1}), where $j=e_{n+1}t+f_{n+1}$ for some e_{n+1}, f_{n+1}. By the Chinese remainder theorem again, we can take K such that $K\equiv f_i'$ (mod e_i) and we have $p_i\nmid 2^K-m$ for $1\le i\le n+1$. Then 2^K-m has at least $n+1$ prime factors $p_1, p_2, \ldots, p_{n+1}$, completing the induction.

Problem 535. Determine all integers $n>4$ such that it is possible to color the vertices of a regular n-sided polygon using at most 6 colors such that any 5 consecutive vertices have distinct colors.

Solution. **CHUI Tsz Fung**, **Hei Chun LEUNG**, **LAU Chung Man** (Lee Kau Yan Memorial School), **LW Maths Solving Team** (SKH Lam Woo Memorial Secondary School) and **Toshihiro SHIMIZU** (Kawasaki, Japan).

Let the colors be *a, b, c, d, e, f.* Denote by S_1 the sequence *a, b, c, d, e* and by S_2 the sequence *a, b, c, d, e, f.* If $n>0$ is representable in the form $5x+6y$ for $x,y\ge 0$, then n satisfies the conditions of the problem: we may place x consecutive S_1 sequences, followed by y consecutive S_2 sequences, around the polygon. Setting y equal to 0, 1, 2, 3 or 4, we find that n may equal any number of the form $5x$, $5x+6$, $5x+12$, $5x+18$ or $5x+24$. The only numbers greater than 4 not of this form are 7, 8, 9, 13, 14 and 19. Below we will show that none of these numbers has the required property.

Assume for a contradiction that a coloring exists for n equal to one of 7, 8, 9, 13, 14 and 19. There exists a number k such that $6k < n < 6(k+1)$. By the pigeonhole principle, at least $k+1$ vertices of the n-gon have the same color. Between any two of these vertices are at least 4 others, because any 5 consecutive vertices have different colors. Hence, there are at least $5k+5$ vertices, and $n\ge 5k+5$. However, this inequality fails for $n = 7, 8, 9, 13, 14, 19$, a contradiction. Hence, a coloring is possible for all $n\ge 5$ except 7, 8, 9, 13, 14 and 19.

Notes on IMO 2019

(Continued from page 1)

Problem 3 is a graph algorithmic problem. The problem is not real hard, but the essential difficulty is hidden by the numbers, students also might find it hard because they do not have the language of graph theory. Namely the graph is connected, with at least three vertices and is not complete, and there is a vertex of odd degree. Then it is possible to find a vertex and apply the operation, and reduce the number of edges by 1, yet maintaining the essential initial conditions. There is no worry of the existence of a cycle, for instance, during the operations. Otherwise the cycle can only be shrunk to a triangle and get stuck. At least a solution is conceivable.

I do not know what to say about problem 2 and 6 (medium and hard geometry problem). Our team did not do too well. It suffices to say, problem 2 may be done by careful angle chasing, while problem 6 is more complicated, but there is a nice and not too complicated complex number solution.

In short, leaders generally agreed that those problems are do-able. If one understands what is going on, one should be able to do those problems, and there is no need of deep and/or obscure theorems. I recalled one of my teachers told us, there really is "no mystery", if you get the point. Also it came to my mind Hilbert's motto: *wir mussen wissen*, *wir werden wussen* (we must know, we will know). Indeed at the end, the cut-off scores were relatively high, 17 for bronze, 24 for silver, and 31 for gold, and in total 6 contestants obtained full mark.

After coordination and the final Jury meeting, we managed to get 1 silver medal (Harris) and 3 bronze (Wan, Daniel and Omega). Surely it was not too good, but not too bad either. Indeed they could do better. For instance, Bruce was only 1 point below bronze, and Sui Kei 3 points (he got a honorable mention by scoring full mark in a problem), should they not making several trivial mistakes (also made by members of several strong teams), they should get medals. Both Daniel and Wan solved three problems, and in my opinion potential silver medalists. On the whole, I notice they have been working hard during the last two months, so I don't think I should blame them too much. One thing however I think our team members should watch out is, in case they will come back next time, they should know how much further effort they need to devote and know what they expect.

I have given my opinions and suggestions. Accordingly 2020 IMO will be held in Russia, 2021 in USA, 2022 in Norway, 2023 in Japan, 2024 in Shanghai China (probably) and 2025 in Australia. Some people have been working hard to make future IMOs possible. I hope Hong Kong will continue to join. However I cannot be too sure. For one thing, not sure if Hong Kong will be as relatively free/peaceful/prosperous to sustain events of this kind. Even so, I am not quite sure if our students may maintain their interest. Life is hard (as usual). Let's hope for the best. Good Luck.