



The Hong Kong University of Science and Technology

Department of Mathematics

Seminar on Applied Mathematics

**Numerical solutions of stochastic time-fractional
PDEs driven by space-time white noise**

by

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Abstract

In this talk, we consider the numerical methods and analysis for the stochastic time-fractional equation

$$\partial_t \psi - \Delta \partial_t^{1-\alpha} \psi = f + \varepsilon \dot{W}$$

in a convex polygon/polyhedron $\mathcal{O} \subset \mathbb{R}^d$, $d \in \{1, 2, 3\}$, where $\partial_t^{1-\alpha} \psi$ (for given $\alpha \in (0, 1) \cup (1, 2)$) denotes the Caputo fractional derivative/integral, $f(x, t)$ a given deterministic source function, ε a given positive parameter, and \dot{W} space-time white noise. Such problems arise naturally by considering the heat transfer in a material with thermal memory, subject to white noise. For the above model, both the time-fractional derivative and the stochastic process result in low regularity of the solution. Hence, the numerical approximation of such problems and the corresponding numerical analysis are very challenging. In this work, the stochastic time-fractional equation is discretized by a backward Euler convolution quadrature in time with piecewise continuous linear finite element method in space. The sharp-order convergence

$$\mathbb{E} \|\psi(\cdot, t_n) - \psi_n^{(h)}\|_{L^2(\mathcal{O})} = \begin{cases} O(\tau^{\frac{1}{2} - \frac{\alpha d}{4}} + \ell_h^{\frac{1}{2}} h^{\frac{1}{\alpha} - \frac{d}{2}}) & \text{if } \alpha \in \left[\frac{1}{2}, \frac{2}{d}\right), \\ O(\tau^{\frac{1}{2} - \frac{\alpha d}{4}} + h^{2 - \frac{d}{2}}) & \text{if } \alpha \in \left(0, \frac{1}{2}\right), \end{cases}$$

up to a logarithmic factor $\ell_h^{\frac{1}{2}} = (\ln(e+1/h))^{\frac{1}{2}}$, is established in general d -dimensional spatial domains, where $\psi_n^{(h)}$ denotes the approximate solution at the n th time step, and \mathbb{E} the expectation operator. Numerical results are presented to illustrate the theoretical analysis.

Date: Tuesday, 7 August 2018

Time: 10:30a.m. – 11:30a.m.

Venue: Room 5508, Academic Buildings (Lifts 25, 26), HKUST

All are welcome!