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The Hong Kong University of Science and Technology
Department of Mathematics

Seminar on Pure Mathematics

The Solution of Wang's Problem

on Permanents of (-1, 1)-Matrices

By

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Abstract

The class of $(-1,1)$-matrices is very important in algebra and combinatorics and in various their applications. For example, well-known Hadamard matrices are of this type.

Two important functions in matrix theory, determinant and permanent, look very similar:

$$\det A = \sum_{\sigma \in S_n} (-1)^{\sigma} a_{1\sigma(1)} \cdots a_{n\sigma(n)} \quad \text{and} \quad \operatorname{per} A = \sum_{\sigma \in S_n} a_{1\sigma(1)} \cdots a_{n\sigma(n)}$$

here $A = (a_{ij}) \in M_n(\mathbb{F})$ is an $n \times n$ matrix and S_n denotes the set of all permutations of the set $\{1, \ldots, n\}$.

While the computation of the determinant can be done in a polynomial time, it is still an open question, if there are such algorithms to compute the permanent.

In this talk we discuss the permanents of ± 1-matrices.

In 1974 Wang [2, Problem 2] posed a problem to find a decent upper bound for $|\operatorname{per}(A)|$ if A is a square ± 1-matrix of rank k. In 1985 Kräuter [1] conjectured some concrete upper bound.

We prove the Kräuter's conjecture and thus obtain the complete answer to the Wang's question. In particular, we characterized matrices with the maximal possible permanent for each value of k.

Date:	**Friday, 07 December 2018**
Time:	**10:00a.m.-11:00a.m.**
Venue:	**Room 4503, Academic Building (Lifts 25 & 26), HKUST**

All are welcome