The Hong Kong University of Science and Technology

Department of Mathematics

<u>Seminar on Pure Mathematics</u>

Recent progress of various Volume Conjectures for links as well as 3-manifolds

By

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Abstract

The original Volume Conjecture of Kashaev-Murakami-Murakami predicts a precise relation between the asymptotics of the colored Jones polynomials of a knot in S^3 and the hyperbolic volume of its complement. I will discuss two different directions that lead to generalizations of this conjecture.

The first direction concerns different quantum invariants of knots, arising from the colored $SU(n)$ (with the colored Jones polynomial corresponding to the case $n= 2$). I will first display subtle relations between congruence relations, cyclotomic expansions and the original Volume Conjecture for colored Jones polynomials of knots. I will then generalize this point of view to the colored $SU(n)$ invariant of knots. Certain congruence relations for colored $SU(n)$ invariants, discovered in joint work with K. Liu, P. Peng and S. Zhu, lead us to formulate cyclotomic expansions and a Volume Conjecture for these colored $SU(n)$ invariants. I will also discuss similar ideas for the superpolynomials that arise in HOMFLY-PT homology. In fact, I proposed cyclotomic expansion conjectures and Volume conjectures for superpolynomials.

Another direction for generalization involves the Witten-Reshetikhin-Turaev and (modified) Turaev-Viro quantum invariants of 3-manifolds. In a joint work with T. Yang, we formulated a new Volume Conjecture for the asymptotics of these 3-manifolds invariants evaluated at certain roots of unit, and numerically checked it for many examples. Interestingly, this conjecture uses roots of unity that are different from the one usually considered in literature. This may indicate that the understanding of this new phenomenon requires new physical and geometric interpretations that go beyond the usual quantum Chern-Simons theory. I will also introduce a work on Krillov-Reshetikhin quantum 6j-symbols done by J. Murakami & me.

Date:	**Monday, 07 January 2019**
Time:	**4:00p.m. - 5:00p.m.**
Venue:	**Room 4475, Academic Building** **HKUST** (*Lifts 25-26*)

All are welcome!