



**The Hong Kong University of Science and Technology**

**Department of Mathematics**

**PhD THESIS EXAMINATION**

***Geometric and algebraic parameterizations for Dirac cohomology of simple modules in  $\mathcal{O}^p$  and their applications***

*By*

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**ABSTRACT**

In this thesis, we show that the Dirac cohomology  $H_D(L(\lambda))$  of a simple highest weight module  $L(\lambda)$  in  $\mathcal{O}^p$  can be parameterized by a specific set of weights: a subset  $\mathcal{W}_l(\lambda)$  of the orbit of the Weyl group  $W$  acting on  $\lambda + \rho$ . As an application, we show that any simple module in  $\mathcal{O}^p$  is determined up to isomorphism by its Dirac cohomology. We describe four parameterizations of  $H_D(L(\lambda))$  when  $\lambda$  is regular. Two of these parameterizations are geometric in terms of a partial ordering on the dual of the Cartan subalgebra and a generalization of strong linkage, respectively. Using these geometric parameterizations, we derive two algebraic parameterizations in terms of the multiplicities of the composition factors of a Verma module and the embeddings between Verma modules, respectively. As an application, for Verma modules with regular infinitesimal character, we obtain an extended version of the Verma-BGG Theorem. We also investigate Dirac cohomology of Kostant modules. Using Dirac cohomology, we give a new proof of the simplicity criterion for Verma modules and describe a new simplicity criterion for parabolic Verma modules with regular infinitesimal character.

**Date: 06 Jun 2019, Thursday**

**Time: 3:00 p.m.**

**Venue: Room 3494 (near lifts 25-26)**

**Thesis Examination Committee:**

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*(Open to all faculty and students)*

The student's thesis is now being displayed on the reception counter in the General Administration Office (Room 3461).