# The Hong Kong University of Science and Technology

## Department of Mathematics

# PhD THESIS EXAMINATION

## *Threshold dynamics: Analysis and Applications*

***By***

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***ABSTRACT***

In this thesis, the threshold dynamics method is extended to a general interfacial energy and applied to topology optimization problem in fluids.

The threshold dynamics was first proposed by Merriman, Bence and Osher to simulate mean curvature flow. It iterates in two steps, solving a heat equation and thresholding at a specific value. Esedoglu and Otto establish a variation formulation of the threshold dynamics, where the algorithm is viewed as iteratively minimizing interfacial energy. This viewpoint extend the threshold dynamics to network with arbitrary surface tensions. However, the surface tension of each interface is limited to a constant and thus in general it can only generate mean curvature motion. We remove this restriction. We design an energy functional and establish its convergence to interfacial energy. Based on this approximate energy functional, we extend the threshold dynamics method to more general cases while preserving all its nice properties. It is simple, efficient and unconditionally energy stable. The proposed method has been applied to many practical problems.

Topological optimization is a tool for engineering structural designs and has been shown to improve the structural performance of materials significantly. Topological optimization seeks to arrange two or more materials in the design domain such that certain objectives are optimized under certain constraints. To find out the distributions of materials is equivalent to find out the interfaces separating them. We adapt threshold dynamics to topology optimization problem in fluid, it is rigorously proven to improve objective functional iteratively. The resulting algorithm alternates solving a state equation and updating interfaces by a threshold-dynamic type step. The proposed method is also efficient, robust and unconditionally energy stable. The stability enables us to design adaptive scheme, which further improves efficiency and makes the method insensitive to initial guess and parameters. This method has been successfully applied to many topology optimization problems in fluid, including Stokes problem, Darcy Stokes problem and biological network problem.


**Date: 30 Nov 2020, Monday**

**Time: 4:30 p.m.**

**Venue: Online via Zoom**

**Zoom ID: 940 8396 1927** (Passcode: 208138)

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***(Open to all faculty and students)***

**The student's thesis is now being displayed on the reception counter in the General Administration Office (Room 3461).**