The Hong Kong University of Science and Technology

Department of Mathematics

PhD THESIS EXAMINATION

Nilpotent orbits and Dynkin indices of Lie algebras

By

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ABSTRACT

We study nilpotent orbits and Dynkin indices of Lie algebras. After a brief review of nilpotent orbit theory, we study a relation between the subregular element and the subregular nilpotent element of a reductive Lie algebra. We make a review about the root systems and representations of $\mathfrak{sl}(3,C)$ and complex exceptional Lie algebra $\mathfrak{g}_2$. Our calculations of orthogonal projections and Dynkin indices are based on $\mathfrak{sl}(3,C)$ and $\mathfrak{g}_2$. We study the orthogonal projections of irreducible root systems to the plane spanned by the root system of type $\mathcal{A}_2$. We get the type $\mathcal{G}_2$ root system for all the root systems except type $\mathcal{C}$. With the above orthogonal projections, we can see the close relation between $\mathfrak{sl}(3,C)$ and $\mathfrak{g}_2$. We study the Dynkin index which is defined by E. Dynkin. Dynkin indices describe the ways of embeddings of a simple subalgebra into a complex simple Lie algebra. We calculate the Dynkin indices of $\mathfrak{sl}(3,C)$-module, $\mathfrak{so}(5,C)$-module, and $\mathfrak{g}_2$-module. We also get a general formula for the Dynkin index of $\mathfrak{sl}(l+1,C)$-module. As a practice of computational Lie theory, we re-calculate the Dynkin indices of subalgebras of type $\mathcal{A}_2$ and type $\mathcal{G}_2$ in the five exceptional simple Lie algebras. We use the computer algebra system *SageMath* to do the branching rules and find maximal subgroups (subalgebras) of the exceptional Lie groups (algebras). In this process, we find a new Dynkin index of $\mathfrak{g}_2$ in $\mathfrak{e}_8$ with value 4. Lastly, we study the Lie algebra $\mathfrak{e}_{7\frac{1}{2}}$. This Lie algebra comes from P. Deligne's work about the exceptional series of Lie groups. We study the structure of $\mathfrak{e}_{7\frac{1}{2}}$ in the frame of parabolic subalgebra of Heisenberg type. After getting the branching rules with *SageMath*, we study the Dynkin indices for $\mathfrak{e}_{7\frac{1}{2}}$.

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The student's thesis is now being displayed on the reception counter in the General Administration Office (Room 3461).