The Hong Kong University of Science and Technology

Department of Mathematics

PhD THESIS EXAMINATION

Holonomic Bessel Modules and Generating Functions

By

Ms. Xiaoli LIN

ABSTRACT

We develop a *D*-module approach to generating functions for Bessel functions in this thesis. By solving a specific system of holonomic PDEs of Bessel modules from the viewpoint of Bernsteins holonomic module theory, we show that the difference Bessel equations and its solutions recently discovered by Bohner and Cuchta and the classical Bessel functions can be unified and derived under this Weyl-algebraic interpretation. We have discovered difference Bessel polynomials from studying the Bessel polynomial module. This Bessel polynomial module allows us to derive the delay-difference formulae and generating functions for the difference Bessel polynomials and classical Bessel polynomials. Then we study the orthogonality of these Bessel polynomials as a residue map which is a left *D*-linear map. In the case of difference Bessel polynomials, the residue map is given by a Barnes integral. Similarly, we construct a *q*-Bessel module base on a *q*-deformation of the commutator set up to study the generating functions of Jacksons *q*-Bessel functions. Finally, we also use a fractional commutator to study and unify different notions of fractional derivatives and fractional differences found in the literatures.

Date : **15 August 2022, Monday**

Time : **4:00 p.m.**

Venue : **Mix Mode**
(Rm 3472, lifts 25-26, 3/F Academic Building, HKUST;
Zoom ID: 970 8321 4792 (Passcode: 758972)
https://hkust.zoom.us/j/97083214792)

***Thesis Examination Committee*:**

Chairman	**:**	**Prof Ping SHENG, PHYS/HKUST**
Thesis Supervisor	**:**	**Prof Yik Man CHIANG, MATH/HKUST**
Member	**:**	**Prof Yongchang ZHU, MATH/HKUST**
Member	**:**	**Prof Ivan Chi Ho IP, MATH/HKUST**
Member	**:**	**Prof Yi WANG, PHYS /HKUST**
External Examiner	**:**	**Prof Erik KOELINK, Department of Mathematics/ Radboud University**

(Open to all faculty and students)

The student's thesis is now being displayed on the reception counter in the General Administration Office (Room 3461).