



The Hong Kong University of Science and Technology

Department of Mathematics

**PhD THESIS EXAMINATION**

**Edge-to-edge Tilings of the Sphere by Congruent Polygons**

By

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**ABSTRACT**

We classify the edge-to-edge tilings of the sphere by congruent polygons. For simplicity, we call them *the tilings*. By the Euler formula, these polygons are triangles, quadrilaterals and pentagons. Duncan Sommerville's 1923 effort commenced the quest of classification for the tilings by congruent triangles. The endeavour eventually came to fruition in 2002 following the work of Yukaku Ueno and Yoshio Agaoka. Various mathematicians have since made significant progress on the other aspects of the subject matter. Notably, Min Yan and his collaborators have largely settled the classification of the tilings by congruent pentagons. Yohji Akama et al. have built preliminaries for the quadrilateral counterpart. We call a polygon *almost equilateral* if all but one of its edges have the same length. The outstanding problems are on the tilings by congruent almost equilateral quadrilaterals, by congruent quadrilaterals with exactly three distinct edges, and by congruent almost equilateral pentagons respectively. They are challenges at a different level and the tools from previous works regrettably fall short. Modern concepts and techniques are developed in this thesis. The advancement enables us to not only resolve the outstanding problems but also simplify the classification for the tilings by congruent triangles. In particular, we attain the tilings by congruent almost equilateral quadrilaterals via two different strategies. These results conclude the classification for the edge-to-edge tilings of the sphere by congruent polygons under a unified framework.

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*(Open to all faculty and students)*

The student's thesis is now being displayed on the reception counter in the General Administration Office (Room 3461).