A fake projective plane (FPP), first constructed by D. Mumford in 1979, is a complex algebraic surface $X$ with Betti numbers $b_i(X) = b_i(\mathbb{P}^2)$ for all $i$. FPPs are known to be arithmetic two-ball quotients. By the work of Prasad-Yeung'07 and '10 and Cartwright-Steger'10, there are 100 FPPs up to biholomorphisms. On the other hand, an FPP is a surface of general type whose algebraic properties are not well understood. For example, Borisov-Keum'20 has found the explicit equations of one specific pair of FPPs only recently. In this talk, we discuss two questions related to FPPs: the first one concerns the existence problem of an explicit type of exceptional collection on the derived category of an FPP by Galkin-Katzarkov-Mellit-Shiner'13; the other one is the constructing problem of surfaces of maximal canonical degree after Beauville'79.