The Hong Kong University of Science and Technology

Department of Mathematics

PhD THESIS EXAMINATION

Embedding Physical Laws into PDEs Solving in Elasticity and Electromagnetics

By

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ABSTRACT

Incorporating physical laws into the solving process of partial differential equations can lead to more efficient algorithms. This thesis presents our exploration of physical modeling and development of algorithms in elastic and electromagnetism equations.

In the first part, we study the 2+1 dimensional continuum model for the evolution of stepped epitaxial surface under long-range elastic interaction. We propose a modified model based on the underlying physics which fixes the problem of illposedness due to the nonconvexity of the energy functional. We prove the existence and uniqueness of both the static and dynamic solutions and derive a minimum energy scaling law for them. We show that the minimum energy surface profile is mainly attained by surfaces with step meandering instability.

In the second part, we propose a neural network structure called energy decay convolutional network (EDCnet) for solving general gradient flow equations. We organize the connection of variables for the PDE based on an energy decay scheme while the corresponding operators are learned by convolutional block. Our method is fast, stable, and can be well generalized to other problems. We applied our method to a wide variety of gradient flow equations to demonstrate its promising applications.

Finally, we propose a singular value decomposition (SVD) based multigrid method for the Helmholtz equation. This method incorporates two key components for efficiency: Krylov subspace and partial SVD. The former decreases the amplification brings from the classical smoothers while the later alleviates the error caused by mismatched eigenvalues between fine and coarse levels. This iterative-direct mixed method is validated by numerical simulations, especially for scenarios when the spectral distribution is relatively poor.

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Venue : Room 5510 (Lifts 25/26)

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(Open to all faculty and students)

The student's thesis is now being displayed on the reception counter in the General Administration Office (Room 3461).