In this talk I will give a brief overview of the deformation theory for anti-self- dual 4-manifolds, and introduce a geometric differential operator that plays a key role. I will then report on some recent progress toward understanding when the cokernel of this operator vanishes, and the connection to a nonlinear PDE from spectral geometry. This is joint work with Rod Gover.