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THE HONG KONG UNIVERSITY OF SCIENCE & TECHNOLOGY

Department of Mathematics

MATHEMATICS COLLOQUIUM

Unusually large components in some critical random graphs

By

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Abstract

In this talk we consider *component sizes* in *random graphs*. In particular, we focus on the simplest model for a random graph, namely the binomial random graph (also called the Erdős and Rényi random graph), which is obtained from the complete graph on n vertices by independently retaining each edge with probability p and deleting it with probability 1-p. This model exhibits an interesting phase transition in the size of its maximal component. By letting p=c/n, with c>0 constant, it happens that: if c<1, then the components are small (of logarithmic size), whereas when c>1 then most vertices are contained in a unique (giant) component and the remaining nodes are contained in tiny clusters (namely, of logarithmic size). Our focus is on the *critical* regime where c=1 (whence p=1/n). In this regime, it is known that a maximal component contains a polynomial (in n) number of vertices; our goal is to estimate the probability of seeing *unusually large* components in this (critical) regime. We illustrate a robust probabilistic methodology to obtain matching upper and lower bounds for the above-mentioned probability. Our argument is simple and relies on three ingredients: (1) an exploration process, which is an algorithm that we use to reveal the connected components and which reduces the study of component sizes to the problem of analyzing the trajectory of a random process; (2) a "ballot-type" estimate, concerning the probability that a random walk stays positive for a given number of steps and finishes at a certain level; (3) and a Brownian motion approximation to random walk. Time permitting, we also briefly mention how the above estimates on the probability of seeing unusually large clusters can be used to analyze a *dynamical* version of the random graph, where edges are resampled in continuous time.

Date : 17 November 2023 (Fri)
Time : 3:00pm – 4:00pm
Venue : Lecture Theater F (Lifts 25/26)

All Are Welcome!