

### THE HONG KONG UNIVERSITY OF SCIENCE & TECHNOLOGY

# **Department of Mathematics**

## SEMINAR ON APPLIED MATHEMATICS

Riemannian optimization and Riemannian Langevin Monte Carlo for PSD fixed rank constraints

By

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#### **Abstract**

Positive semi-definite (PSD) fixed rank constraint arises in certain machine learning and data science applications, e.g., a distance matrix. For optimization under such constraints, we study and compare three methodologies for minimizing f(X) with X being a Hermitian PSD fixed rank matrix. The first approach is the simplest factor-based Burer-Monteiro method. The second approach is to regard the set of Hermitian PSD fixed rank matrices as an embedded manifold in the Euclidean space and consider the Riemannian optimization over the embedded manifold. The third approach is to regard it as a quotient manifold and consider the quotient manifold with a nonflat metric. We show that CG in the first two methodologies is equivalent to CG on the quotient manifold with suitably chosen metrics, retractions, and vector transports. The simple Burer-Monteiro approach corresponds to the Bures-Wasserstein metric. We also analyze the condition number of the Riemannian Hessian under these different metrics. The difference in the condition number of the Riemannian Hessian under different metrics is consistent with the difference in the numerical performance of three methodologies for problems including matrix completion, phase retrieval, and interferometry recovery. This part is based on joint work with Shixin Zheng at Purdue University, Wen Huang at Xiamen University and Bart Vandereycken at University of Geneva.

Date: 7 December 2023 (Thursday)

Time: 10:00am - 11:00am

Venue: Room 4475 (Lifts 25/26)

All are Welcome!