



THE HONG KONG UNIVERSITY OF SCIENCE & TECHNOLOGY

Department of Mathematics

SEMINAR ON STATISTICS

The second–derivative lower–bound function (SeLF) algorithm and three acceleration techniques for maximization with strongly stable convergence

By

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Abstract

In this paper, we propose a new maximization method, called as the *second-derivative lower-bound function* (SeLF) algorithm. The SeLF algorithm is a general principle for iteratively calculating the MLE $\hat{\theta}$ of the parameter θ in a one-dimensional target function (usually, the marginal log-likelihood function) $l(\theta)$ and its each iteration consists of two steps: A second-derivative lower-bound function step (SeLF-step) and a maximization step (M-step), where the SeLF-step firstly finds a function $b(\theta)$ satisfying $l''(\theta) \geq b(\theta)$ for all $\theta \in \Theta$ and secondly constructs a surrogate function $Q(\theta|\theta^{(t)})$ [whose form depends on $\theta^{(t)}$ being the t -th iteration of $\hat{\theta}$] minorizing $l(\theta)$ at $\theta = \theta^{(t)}$, and the M-step calculates the maximizer $\theta^{(t+1)}$ of the $Q(\theta|\theta^{(t)})$ function, which is equivalent to solving the equation $l'(\theta) + \int_{\theta^{(t)}}^{\theta} b(z) dz = 0$ to obtain its explicit solution $\theta^{(t+1)}$. The SeLF algorithm holds two major advantages: (i) It strongly stably converges to the MLE $\hat{\theta}$, in contrast to general *minorization-maximization* (MM) algorithms only possessing weakly stable convergence; and (ii) it does not depend on any initial values, in contrast to Newton's method. The key for applying the SeLF algorithm is to find a function $b(\theta)$ satisfying $l''(\theta) \geq b(\theta)$ for all $\theta \in \Theta$ such that an explicit solution to the equation $l'(\theta) + \int_{\theta^{(t)}}^{\theta} b(z) dz = 0$ is available. Furthermore, we develop three acceleration techniques (i.e., optimal SeLF, sub-optimal SeLF, and fast-SeLF algorithms) for the SeLF algorithm, resulting in a weakly stable convergence. Various applications in statistics of the proposed SeLF algorithm and three acceleration versions are introduced. The analysis of the convergence rates of these algorithms are provided. Some numerical experiments and comparisons are also presented. [This is a joint work with Dr. Xunjian LI]

Biography

Dr. Guo-Liang Tian has been a Full Professor of Statistics at Department of Statistics and Data Science of Southern University of Science and Technology (SUSTech) since 2016. He was an Associate Professor of Statistics at Department of Statistics and Actuarial Science of the University of Hong Kong from 1998 to 2016. His current research interests include EM/MM/US algorithms with applications in statistics, statistical analyses of multivariate continuous proportional data in m -dimensional cubic $(0, 1)^m$, and statistical analysis of multivariate zero-inflated count data. He was the author of 16 top tier biostatistics papers and more than 140 other statistics papers in peer-reviewed international academic journals. He published three statistical monographs and two statistical textbooks in Science Press. His two grants were funded by the National Natural Science Foundation of China.

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All are Welcome!