The Hong Kong University of Science and Technology

Department of Mathematics

<u>PhD THESIS EXAMINATION</u>

Spectral Theory of Covariance Matrices with Its Statistical Applications

By

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<u>ABSTRACT</u>

As one of the most classic models in random matrix theory, the sample covariance matrices have been widely studied. When considering the high-dimensional setting it is well-known that the empirical spectral distribution converges to Marchenko-Pastur law (MP law). Inspired by problems such as Principal Component Analysis (PCA), the extreme eigenvalue has also been extensively studied. This motivates our first part of the thesis, focusing on the extreme eigenvalues of sample covariance matrices for two distinct ensembles: the log-concave ensemble and the heavy-tailed ensemble. Our investigations yield significant insights into the Tracy-Widom law for extreme eigenvalues and unveil a phase transition phenomenon induced by the tail fatness of matrix entries. In addition to advancing the spectral theory of sample covariance itself, tools from random matrix theory find application in various statistical problems. One notable example is the regularized Ridge estimators for linear regression models. In the second part of the thesis, we explore the distributional characterization of Ridge estimators. By employing both a convex Gaussian min-max theorem approach and an approximate message passing strategy, we achieve a comprehensive understanding of the stochastic behaviors of Ridge estimators across various scenarios. Our analyses shed light on the statistical properties of Ridge estimators in diverse tasks, including prediction accuracy and statistical inference.

Date : 20 May 2024, Monday

Time : 3:00 pm

Venue : Room 4472 (Lifts 25/26)

<u>Thesis Examination Committee</u>:

Chairman	:	**Prof. Hai YANG, CIVL/HKUST**
Thesis Supervisor	:	**Prof. Zhigang BAO, MATH/HKUST**
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(Open to all faculty and students)

The student's thesis is now being displayed on the reception counter in the General Administration Office (Room 3461).