



THE HONG KONG UNIVERSITY OF SCIENCE & TECHNOLOGY

Department of Mathematics

SEMINAR ON APPLIED MATHEMATICS

HJ-sampler: a Bayesian Sampler for Inverse Problems of a Stochastic Process by Leveraging Hamilton--Jacobi PDEs and Score-based Generative Models

By

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Abstract

The interplay between stochastic processes and optimal control has been extensively explored in the literature. With the recent surge in the use of diffusion models, stochastic processes have increasingly been applied to sample generation. This paper builds on the log transform, known as the Cole-Hopf transform in Brownian motion contexts, and extends it within a more abstract framework that includes a linear operator. Within this framework, we found that the well-known relationship between the Cole-Hopf transform and optimal transport is a particular instance where the linear operator acts as the infinitesimal generator of a stochastic process. We also introduce a novel scenario where the linear operator is the adjoint of the generator, linking to Bayesian inference under specific initial and terminal conditions. Leveraging this theoretical foundation, we develop a new algorithm, named the HJ-sampler, for Bayesian inference for the inverse problem of a stochastic differential equation with given terminal observations. The HJ-sampler involves two stages: solving viscous Hamilton-Jacobi (HJ) partial differential equations (PDEs) and sampling from the associated stochastic optimal control problem. Our proposed algorithm naturally allows for flexibility in selecting the numerical solver for viscous HJ PDEs. We introduce two variants of the solver: the Riccati-HJ-sampler, based on the Riccati method, and the SGM-HJ-sampler, which utilizes diffusion models. Numerical examples demonstrate the effectiveness of our proposed methods. This is an ongoing joint work with Zongren Zou, Jerome Darbon, and George Em Karniadakis.

Date : 13 June 2024 (Thursday)

Time : 10:00a.m.-11:00a.m.

Venue : Room 2463 (near Lift 25/26)

All are Welcome!