The Hong Kong University of Science and Technology

Department of Mathematics

PhD THESIS EXAMINATION

The Diffeomorphic Embedding Method for Hyperbolic Conservation Laws on Implicit Surfaces

By

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ABSTRACT

In this thesis, a novel embedding method called the diffeomorphic embedding method is developed for solving scalar hyperbolic conservation laws posed on implicit surfaces. The surface is represented implicitly as a zero-level set of the signed distance function, from which many geometric quantities can be easily computed. As in other embedding methods, an embedding partial differential equation (PDE) needs to be posed in a tubular neighborhood of the surface, and the solution to this equation should resemble the solution to the original surface equation. By inspecting embedding methods from a differential geometric viewpoint, the author introduces the push-forward operator into the construction of the embedding PDEs. Theoretically, the solution is guaranteed to be constant along the surface normal directions, and high-order accuracy can be achieved while capturing the discontinuities. The push-forward operator naturally extends any tangential vector from the surface to all nearby level surfaces. A simple formula involves only the signed distance function and its Hessian matrix is provided, and all computation is done in Cartesian coordinates. Various numerical experiments are performed to demonstrate the accuracy and efficiency of the proposed scheme. The proposed method is named the diffeomorphic embedding method to honor the importance of diffeomorphism in the design. This method addresses the gap in solving hyperbolic conservation laws on surfaces by embedding methods and offers a unique perspective on understanding embedding methods.

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Time : 3:00 pm

Venue : Room 5506 (Lifts 25-26)

***Thesis Examination Committee*:**

Chairman	**:**	**Prof. Hong Kam LO, CIVL/HKUST**
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(Open to all faculty and students)

The student's thesis is now being displayed on the reception counter in the General Administration Office (Room 3461).