# The Hong Kong University of Science and Technology

## Department of Mathematics

# PhD THESIS EXAMINATION

## Mathematics of Super-resolution: Theory and Algorithms

*By*

## Mr. Zetao FEI


***ABSTRACT***

Appearing in different literature, super-resolution mainly refers to the techniques that enhance the resolution of signals. Since the birth of the microscope, super-resolution has been a central problem for imaging systems for about three centuries. In this thesis, we consider the mathematics of super-resolution problem from both theoretical and algorithmic perspectives. For closely positioned point sources, we develop the Iterative Focusing-localization and Filtering (IFF) algorithm for super-resolution problem with multiple Fourier measurements. We quantitatively analyze its resolution limit and numerical experiments show that the algorithm can achieve stable reconstruction near the theoretical resolution limit. For sources widely spread over the real line, we propose a scalable efficient algorithm called SCAN-MUSIC. The proposed algorithm achieves optimal sampling complexity. When compared with the state-of-the-art algorithm, the proposed one has comparable efficiency while having unique strength for clustered sources. Finally, we go beyond the point source model and propose a unified framework of super-resolution (Model-SR). We show that under suitable modeling, the resolution enhancing map has Lipschitz stability. The proposed mathematical framework can be extended to problems with similar structures.


**Date : 29 July 2024, Monday**

**Time : 3:00 pm**

**Venue : Room 5510 (Lifts 25-26)**

***Thesis Examination Committee*:**

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| **Chairman** | **:** | **Prof. Iam Keong SOU, PHYS/HKUST** |
| **Thesis Supervisor** | **:** | **Prof. Hai ZHANG, MATH/HKUST** |
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***(Open to all faculty and students)***

**The student's thesis is now being displayed on the reception counter in the General Administration Office (Room 3461).**