



**The Hong Kong University of Science and Technology**

**Department of Mathematics**

**MPhil THESIS EXAMINATION**

# **On the Category of Singularities**

*By*

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**ABSTRACT**

We can attach to any scheme  $X$  the category of singularities of  $X$  which is trivial when  $X$  is smooth. Thus, in some sense, it captures information about the singularities of the scheme. Orlov showed that if  $f: X \rightarrow A^1$  is a morphism and  $X$  is a regular scheme, then the category of the singularities of the zero locus of  $f$  is equivalent to the category of matrix factorizations of  $(X, f)$ , a more explicit category. This is a particular instance of the so-called Koszul duality for curved complexes by Positselski that seems to be understood by a small number of experts.

This thesis is motivated by the desire to interpret and reprove Orlov's result in the modern language of stable infinity categories, whose first step would involve a construction of the category of matrix factorizations in a homotopically correct way from the ground up. More precisely, whereas previous authors started with a point-set model using explicit (curved) chain complexes, followed by a localization procedure, we hope to start within the world of stable infinity categories already, from which the category of matrix factorizations is obtained via general categorical algebra methods.

From this perspective, this thesis provides the first step toward this goal: we provide a new way to understand complexes (as opposed to curved complexes) within the world of stable infinity categories. More concretely, we answer the questions: (1) what does it mean to rigidify a chain complex in a derived category so that the differential  $d$  becomes visible, and (2) how does one “forget” this differential using categorical algebras? This is a warm-up to the case of curved complexes where we need to talk about  $d^2 = c$  for some curvature  $c$ .

**Date : 9 August 2024, Friday**

**Time : 4:00 p.m.**

**Venue : Room 5501 (Lifts 25-26)**

**Thesis Examination Committee**

**Chairman : Prof. Ivan Chi Ho IP, MATH /HKUST**

**Thesis Supervisor : Prof. Quoc HO, MATH/HKUST**

**Member : Prof. Qingyuan JIANG, MATH/HKUST**

*(Open to all faculty and students)*

The student's thesis is now being displayed on the reception counter in the General Administration Office (Room 3461).