The Hong Kong University of Science and Technology

Department of Mathematics

PhD THESIS EXAMINATION

Efficient Algorithms for Nonconvex Optimization with Applications in Proton Therapy and Distributionally Robust Optimization

By

Mr. Junyi FAN

ABSTRACT

Efficient algorithms in nonconvex optimization can significantly improve accuracy, speed, and scalability in various fields. This thesis presents our development of such algorithms, which have the potential to greatly influence many applications. The first part of this thesis focuses on the nonconvex Minimum-monitor-unit (MMU) constraint in pencil beam scanning proton therapy, crucial for treatments like efficient IMPT, FLASH RT, and proton arc (ARC) delivery. Existing methods struggle with large MMU thresholds, so we propose the stochastic coordinate descent (SCD) method. SCD ensures a monotonic decrease in the optimization objective and provides explicit updates for each coordinate. Numerical experiments on three clinical cases show that SCD improves treatment plan quality with large MMU thresholds in both IMPT and ARC. We also introduce a rigorous spot reduction (RSR) method to enhance delivery efficiency and plan quality. The RSR method formulates the constrained IMPT problem with MMU and ℓ_0 norm constraints and solves it efficiently using the ADMM method. Numerical experiments demonstrate that the RSR method achieves comparable plan quality to SCD but in faster manner. The second part studies the Bures-Wasserstein (BW) gradient descent method, useful in areas like Gaussian barycenter, matrix recovery, and variational inference. Existing convergence analyses often depend on specific loss functions and lack exploration of constrained settings. We address this by providing a general convergence rate guarantee for BW gradient descent under Euclidean strong convexity of the loss and constraints. We also derive a closed-form solution for projection onto BW distance-constrained sets, enabling fast implementation of projected BW gradient descent for constrained barycenter and distributionally robust optimization problems. Experimental results show significant improvements in computational efficiency and convergence speed, highlighting our method's effectiveness in practical scenarios.

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Time : 3:00 pm

Venue : Room 4475 (Lifts 25-26)

Thesis Examination Committee:

Chairman	:	**Prof. Wenjing YE, MAE/HKUST**
Thesis Supervisor	:	**Prof. Jianfeng CAI, MATH/HKUST**
Member	:	**Prof. Dong XIA, MATH/HKUST**
Member	:	**Prof. Yang XIANG, MATH/HKUST**
Member	:	**Prof. Yi CHEN, IEDA /HKUST**
External Examiner	:	**Prof. Ke YIN, Huazhong University of Science & Technology**

(Open to all faculty and students)

The student's thesis is now being displayed on the reception counter in the General Administration Office (Room 3461).