



**THE HONG KONG UNIVERSITY OF SCIENCE & TECHNOLOGY**

**Department of Mathematics**

# **SEMINAR ON APPLIED MATHEMATICS**

## **A Journey into Bound-Preserving Schemes and Theory**

**By**

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### **Abstract**

Solutions to many partial differential equations (PDEs) are subject to certain bounds or constraints. For instance, in fluid dynamics, density and pressure must remain positive, while in relativistic cases, fluid velocity must not exceed the speed of light. Developing bound-preserving numerical methods that uphold these intrinsic constraints is crucial. Recently, significant attention has been given to design provably bound-preserving schemes, though challenges remain, particularly for systems with nonlinear constraints.

In this talk, I will present our recent efforts in developing fundamental bound-preserving theories:

1. **Geometric Quasilinearization (GQL):** Drawing on key insights from geometry, we propose a novel and general framework called geometric quasilinearization. GQL offers an effective approach for addressing bound-preserving problems with nonlinear constraints by transforming these constraints into linear ones through the introduction of auxiliary variables. We establish the fundamental principles and general theory of GQL using the geometric properties of convex regions and present three effective methods for constructing GQL.
2. **Optimal Cell Average Decomposition (OCAD):** Utilizing convex geometry and symmetric group theory, we develop the optimal cell average decomposition theory, which provides a foundation for constructing more efficient bound-preserving schemes. We demonstrate that the classic Zhang-Shu CAD is optimal in one dimension but generally not in multiple dimensions, thereby addressing their conjecture proposed in 2010.

We apply the GQL and OCAD approaches to various PDEs, showcasing their effectiveness and advantages through diverse and challenging examples and applications, including magnetohydrodynamics (MHD), relativistic hydrodynamics, and the ten-moment Gaussian closure system.

**Date : 19 August 2024 (Monday)**

**Time : 3:00pm - 4:00pm**

**Venue : Room 4502 (Lifts 25/26)**

*All are Welcome!*