## ALGEBRA AND GEOMETRY SEMINAR The Hong Kong University of Science and Technology Department of Mathematics

## Lecture series: On the theory of double $\infty$ -categories

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This series of three talks aims to give a detailed introduction to double  $\infty$ -categories. Double  $\infty$ -categories can be viewed as generalizations of  $(\infty, 2)$ -categories that admit two directions for morphisms. The series starts by motivating these categorical constructions, and we will see how these appear in mathematics. We will discuss their definitions, including different completeness assumptions. Moreover, we will see how double  $\infty$ -categories can be used to model  $(\infty, 2)$ -categories. We highlight some important examples of double  $\infty$ -categories throughout.

We will then continue to study the notions of companionships and conjunctions in double  $\infty$ -categories. These are important and useful concepts that can be used to describe the universal property of socalled squares constructions, as we will see. Moreover, we will study functors between double  $\infty$ -categories and show that they assemble into double  $\infty$ -categories of functors with vertical and horizontal natural transformations. We present a new result that characterizes the companions and conjoints in these functor double  $\infty$ -categories. On the way, we will see how this double categorical machinery can be specialized to prove results in ( $\infty$ , 2)-category theory.

During the first talk, we will recall some relevant background material on  $\infty$ -categories that will be needed to follow the series. No knowledge of  $(\infty, 2)$ -categories is assumed.

I	Room	4475	(Lifts	25/26)	Wed,	Nov	13,	2024	04:45	ΡM
II	Room	4475	(Lifts	25/26)	Fri,	Nov	15,	2024	05:00	ΡM
III	Room	4475	(Lifts	25/26)	Wed,	Nov	20,	2024	04:45	ΡM

