



THE HONG KONG UNIVERSITY OF SCIENCE & TECHNOLOGY

Department of Mathematics

SEMINAR ON APPLIED MATHEMATICS

**Nehari manifold optimization and its
application for finding unstable solutions
of semilinear elliptic PDEs**

by

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Abstract

A Nehari manifold optimization method (NMOM) is introduced for finding 1-saddles, i.e., unstable saddle points with the Morse index equal to 1 of a generic nonlinear functional in Hilbert spaces. Actually, it is based on the variational characterization that 1-saddles are local minimizers of the generic functional restricted on the associated Nehari manifold. The framework contains two important ingredients: one is the retraction mapping to make the iteration points always lie on the Nehari manifold; the other one is the tangential search direction to decrease the generic functional with suitable step-size search rules. Particularly, the global convergence is rigorously established by virtue of some crucial analysis techniques (including a weak convergence method) overcoming difficulties in the infinite-dimensional setting. In practice, combining with an easy-to-implement Nehari retraction and the negative Riemannian gradient direction, the NMOM is successfully applied to compute the unstable ground-state solutions of a class of typical semilinear elliptic PDEs such as the Henon equation and nonlinear Schrodinger equation. In particular, the symmetry-breaking phenomenon of the ground states of the Henon equation is explored numerically in 1D and 2D with interesting numerical findings on the critical value of symmetry-breaking reported.

Date : 12 February 2025 (Wednesday)

Time : 2:00p.m.-3:00p.m.

Venue : Room 4472 (Lift 25/26)

All are Welcome!