ALGEBRA AND GEOMETRY SEMINAR

The Hong Kong University of Science and Technology
Department of Mathematics

Nonsemisimple Topological Quantum Computation

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Since the foundational work of Freedman, Kitaev, Larsen, and Wang, it has been understood that 3-dimensional topological quantum field theories (TQFTs), described via modular tensor categories, provide a universal model for fault-tolerant topological quantum computation. These TQFTs, derived from quantum groups at roots of unity, achieve modularity by semisimplifying their representation categories—discarding objects with quantum trace zero. The resulting semisimple categories describe anyons whose braiding enables robust quantum computation.

This talk explores recent advances in low-dimensional topology, focusing on the use of nonsemisimple categories that retain quantum trace zero objects to construct new TQFTs. These nonsemisimple TQFTs surpass their semisimple counterparts, distinguishing topological features inaccessible to the latter. For physical applications, unitarity is essential, ensuring Hom spaces form Hilbert spaces. We present joint work with Nathan Geer, Bertrand Patureau-Mirand, and Joshua Sussan, where nonsemisimple TQFTs are equipped with a Hermitian structure. This framework introduces Hilbert spaces with possibly indefinite metrics, presenting new challenges.

We further discuss collaborative work with Sung Kim, Filippo Iulianelli, and Sussan, demonstrating that nonsemisimple TQFTs enable universal quantum computation at roots of unity where semisimple theories fail. Specifically, we show how Ising anyons within this framework achieve universality through braiding alone. The resulting braiding operations are deeply connected to the Lawrence-Krammer-Bigelow representations, with the Hermitian structure providing a nondegenerate inner product grounded in quantum algebra.

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