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THE HONG KONG UNIVERSITY OF SCIENCE & TECHNOLOGY

Department of Mathematics

MATHEMATICS COLLOQUIUM

A new look at quantum knot invariants with an eye toward categorification

By

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Abstract

Quantum knot invariants are powerful tools arising from the interplay of topology and Lie theory. The Jones polynomial, the most famous example, is rooted in the representation theory of the Lie algebra $sl(2)$ of traceless 2×2 matrices. These invariants are fascinating not only because they can distinguish different knots and links but also because they connect to a broad range of mathematical and physical ideas. They give rise to invariants of 3-dimensional manifolds, play a role in exactly solvable models in physics, and describe the braiding of quasiparticle excitations in theories of topological quantum computation.

In this talk, we will explore the work of Cautis, Kamnitzer, and Morrison, who developed a novel diagrammatic framework for constructing quantum knot invariants associated with $sl(n)$. This approach is reminiscent of Kauffman's bracket polynomial for the Jones polynomial, providing an elementary and visual interpretation that avoids the technical complexities of traditional representation theory. By combining two simple Lie-theoretic ingredients with the elegant symmetry of Howe duality, this method offers fresh insights into quantum invariants and their relationships to skein-theoretic constructions.

What makes this perspective especially exciting is its connection to the idea of categorification. The two key Lie-theoretic ingredients in this framework serve as simplified "shadows" of deeper mathematical structures. These richer structures can be categorified, giving rise to more sophisticated invariants known as link homologies, such as Khovanov homology. We will discuss how this new approach provides a direct path to categorifying the entire story, illuminating the interplay between diagrammatic knot invariants and their categorified counterparts.

This talk will not assume any familiarity with representation theory or knot theory and is intended for mathematicians of all backgrounds.

Date : 28 February 2025 (Fri)

Time : 3:00p.m.-4:00p.m.

Venue : Lecture Theatre F (Lift 25/26)

All Are Welcome!