



**THE HONGKONG UNIVERSITY OF SCIENCE & TECHNOLOGY**

**Department of Mathematics**

**SEMINAR ON APPLIED MATHEMATICS**

**Noise-expansion cascade – a fundamental property of  
Navier-Stokes equation**

**By**

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**Abstract**

Randomness is one of the most important characteristics of turbulence, but its origin remains an open question. By means of a “thought experiment” via several clean numerical experiments based on the Navier-Stokes (NS) equations for two-dimensional turbulent Kolmogorov flow, we reveal a new phenomenon, which we call the noise-expansion cascade whereby all micro-level noises/disturbances at different orders of magnitudes in the initial condition of NS equations enlarge consistently, say, one by one like an inverse cascade, to macro-level. More importantly, each noise/disturbance input may greatly change the macro-level characteristics and statistics of the resulting turbulence, clearly indicating that micro-level noise/disturbance might have great influence on macro-level characteristics and statistics of turbulence. Besides, the noise-expansion cascade closely connects randomness of micro-level noise/disturbance and macro-level disorder of turbulence, thus revealing an origin of randomness of turbulence. This also highly suggests that unavoidable thermal fluctuations must be considered when simulating turbulence, even if such fluctuations are several orders of magnitudes smaller than other external environmental disturbances. But, the NS equations completely neglect such kind of small disturbances. Thus, the noise-expansion cascade as a fundamental property of the NS equations might possibly reveal a paradox of the NS equations, the fourth millennium problem posed by Clay Mathematics Institute in 2000.

**BIO:** *Dr. Shijun Liao is a distinguished chair professor of Shanghai Jiaotong University, and the head of State Key Lab of Ocean Engineering. He is the founder of the homotopy analysis method (HAM) for analytic approximation of highly nonlinear problems and the clean numerical simulation (CNS) for long-term reliable simulation of chaos/turbulence. He has published three monographs and more than 212 peer-reviewed articles with the SCI citation 14110 times. He is now an associate editor of Journal of Fluid Mechanics.*

**Date : 07 May 2025 (Wednesday)**

**Time : 4:00p.m.-5:00p.m.**

**Venue : Room 2303 (Lift 17/18)**

*All are Welcome!*