ALGEBRA AND GEOMETRY SEMINAR

The Hong Kong University of Science and Technology
Department of Mathematics

High-rank motivic degree-zero Donaldson–Thomas theory on singular curves, and q-series

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My main point is that high-rank motivic degree-zero DT invariants on singular curves appear to give infinite products of Rogers–Ramanujan type. This is based on explicit computation of certain Quot schemes, which is where the new ideas and results lie, but this seems to be a new phenomenon that I cannot explain from physics or other conceptual connection. For context, the rank-1 case has been observed to relate to knot theory and Catalan combinatorics in the last decade (keyword: Oblomkov–Rasmussen–Shende conjecture).

A down-to-earth statement that captures all the essence is the following (stated for the singular curve $y^2 = x^3$): For a random $n \times n$ matrix A over a finite field $\mathbb{F}_q$, what is the expected number of matrices B such that $AB = BA$ and $A^3 = B^2$? It turns out that as $n \to \infty$, the limiting answer is $\prod(1 - q^{-i})$ over all positive i congruent to $1, 4$ mod 5, the famous Rogers–Ramanujan infinite product.

The reported results contain joint work with Ruofan Jiang (on the $y^2 = x^n$ case) and joint work in progress with RJ and Alexei Oblomkov (on the $y^m = x^n$ case with m, n coprime).

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