



**The Hong Kong University of Science and Technology**

**Department of Mathematics**

**PhD THESIS EXAMINATION**

**Numerical Methods for Differential Equations on Surfaces**

*By*

**Mr. Young Kyu LEE**

**ABSTRACT**

In this thesis, we present two high-order numerical methods for differential equations on surfaces. In the first part, we introduce an embedding method for solving the Laplace-Beltrami eigenproblem on implicitly defined surfaces. By replacing the surface differential operator in the previous work [1] with a novel formulation and incorporating an extension layer, the method achieves second-order accuracy, surpassing the previous approach, while significantly reducing the computational cost by reducing the size of the computational domain from  $O(1)$  to  $O(\Delta x)$ . We also introduce the Motion-DNA, which is a key application of our new approach to distinguish the shapes that share identical static spectra. In the second part, we introduce the Spherical Runge-Kutta methods with Richardson Extrapolation (SRKRE) schemes, which is a class of high-order schemes for solving differential equations on the unit sphere. By adapting Richardson-Extrapolation intrinsically to the sphere's geometry, the method ensures all solutions lie exactly on the unit sphere without additional projection. By combining existing low-order spherical integrators from our previous work [2] with this novel extrapolation mechanism, SRKRE schemes achieve higher-order accuracy and superior performance.

**Date : 16 July 2025, Wednesday**

**Time : 3:00pm**

**Venue : Room 4472 (Lifts 25-26)**

**Thesis Examination Committee:**

<b>Chairman</b>	<b>:</b>	<b>Prof. Pak Wo LEUNG, PHYS/HKUST</b>
<b>Thesis Supervisor</b>	<b>:</b>	<b>Prof. Shing Yu LEUNG, MATH/HKUST</b>
<b>Member</b>	<b>:</b>	<b>Prof. Yang XIANG, MATH/HKUST</b>
<b>Member</b>	<b>:</b>	<b>Prof. Kun XU, MATH/HKUST</b>
<b>Member</b>	<b>:</b>	<b>Prof. Julian MAK, OCS/HKUST</b>
<b>External Examiner</b>	<b>:</b>	<b>Prof. Ronald Lok Ming LUI, The Chinese University of Hong Kong</b>

*(Open to all faculty and students)*

The student's thesis is now being displayed on the reception counter in the General Administration Office (Room 3461).