The Hong Kong University of Science and Technology

Department of Mathematics

<u>PhD THESIS EXAMINATION</u>

Efficient Riemannian Optimization for Low-Rank Matrices: Achieving Optimal Complexity and Robustness to Ill-Conditioning

By

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ABSTRACT

This thesis develops efficient Riemannian optimization algorithms for low-rank matrix problems, with applications in quantum state tomography (QST) and large-scale machine learning. The first part addresses measurement complexity for QST. Convex methods offer near-optimal measurement complexity but suffer from prohibitive computational costs, whereas faster non-convex methods have suboptimal measurement complexity. To bridge this gap, we introduce a novel variant of Riemannian Gradient Descent (RGD). This algorithm provably achieves a measurement complexity of $\mathcal{O}(\mathrm{d}r\kappa^2 \log^6 \mathrm{d})$, the strongest known bound among non-convex QST approaches. Furthermore, it converges linearly at a rate independent of the condition number. The second part addresses computational efficiency for finding low-rank weights in deep neural networks. While existing methods such as Low-Rank Adaptations (LoRA) and RGD offer efficient parameterizations, they often struggle with highly correlated, ill-conditioned real-world inputs. To resolve this, we propose a stochastic version of RGD that operates directly on the manifold of fixed-rank matrices, bypassing the redundancy of factorized representations. By incorporating stochastic gradients, our method maintains low per-iteration cost while naturally mitigating the adverse effects of ill-conditioned inputs. We provide the first theoretical preasymptotic convergence analysis for this approach, proving that the stochastic Riemannian algorithm achieves a convergence rate independent of the condition number of the input data for low-frequency error components. Together, these contributions demonstrate that carefully designed Riemannian gradient methods and their stochastic variants can achieve state-of-the-art efficiency for low-rank matrix problems.

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Time : 10:00 am

Venue : Rm 1409 (Lifts 25/26), 1/F Academic Building, HKUST

<u>Thesis Examination Committee</u>:

Chairman	:	**Prof. Yusong GUO, LIFS/HKUST**
Thesis Supervisor	:	**Prof. Jianfeng CAI, MATH/HKUST** **Prof. Ke WANG, MATH/HKUST**
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(Open to all faculty and students)

The student's thesis is now being displayed on the reception counter in the General Administration Office (Room 3461).