The Hong Kong University of Science and Technology

Department of Mathematics

PhD THESIS EXAMINATION

On the Learning of Gaussian Mixture Models: Model Selection and Parameter Estimation

By

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ABSTRACT

Gaussian Mixture Models (GMMs) are widely utilized for modeling data originating from diverse sources and sub-populations. While maximum likelihood estimation, primarily implemented via the Expectation-Maximization (EM) algorithm, remains the contemporary standard, it often suffers from computational bottlenecks and depends on the initializations, particularly in high-dimensional settings.

In this thesis, we propose a new framework for learning GMMs based on the Fourier measurements of the data. Specifically focusing on parameter estimation and model order selection for GMMs sharing a unified variance or covariance structure, we establish information-theoretic lower and upper bounds on the sample complexity in both univariate and multi-dimensional spaces. By leveraging moment-matching techniques alongside our proposed Fourier approach, we quantify the thresholds of statistical indistinguishability. We develop a suite of efficient estimation algorithms. For high-dimensional parameter estimation, we introduce a gradient-descent-based scheme enhanced by a score-based initialization strategy. Furthermore, by exploiting the underlying algebraic and spectral properties of the mixtures, we propose a specialized, MUSIC-inspired algorithm for GMMs whose component means reside within low-dimensional subspaces. This spectral approach achieves a time complexity linear with respect to the sample size.

Date : 27 May 2026, Wednesday

Time : 10:00 am

Venue : Room 1409 (near Lifts 25/26)

***Thesis Examination Committee*:**

Chairman	**:**	**Prof. Wai Ho MOW, ECE/HKUST**
Thesis Supervisor	**:**	**Prof. Hai ZHANG, MATH/HKUST**
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(Open to all faculty and students)

The student's thesis is now being displayed on the reception counter in the General Administration Office (Room 3461).