



THE HONG KONG UNIVERSITY OF SCIENCE & TECHNOLOGY

Department of Mathematics

SEMINAR ON APPLIED MATHEMATICS

A Fast Butterfly-compressed Hadamard-Babich Integrator for High-frequency Helmholtz Equations in Inhomogeneous Media

By

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Abstract

We present a butterfly-compressed representation of the Hadamard-Babich (HB) ansatz for the Green's function of the high-frequency Helmholtz equation in smooth inhomogeneous media. For a computational domain discretized with N discretization cells, the proposed algorithm first solves and tabulates the phase and HB coefficients via eikonal and transport equations with observation points and point sources located at the Chebyshev nodes using a set of much coarser computation grids, and then butterfly compresses the resulting HB interactions from all N cell centers to each other. The overall CPU time and memory requirement scale as $\mathcal{O}(N \log^2 N)$ for any bounded 2D domains with arbitrary excitation sources. A direct extension of this scheme to bounded 3D domains yields an $\mathcal{O}(N^{\{4/3\}})$ CPU complexity, which can be further reduced to quasi-linear complexities with proposed remedies. The scheme can also efficiently handle scattering problems involving inclusions in inhomogeneous media. Although the current construction of our HB integrator does not accommodate caustics, the resulting HB integrator itself can be applied to certain sources, such as concave-shaped sources, to produce caustic effects. Compared to finite-difference frequency-domain (FDFD) methods, the proposed HB integrator is free of numerical dispersion and requires fewer discretization points per wavelength. As a result, it can solve wave-propagation problems well beyond the capability of existing solvers.

Date : 8 June 2026 (Monday)
Time : 3:00p.m.-4:00p.m.
Venue : Room 2463 (Lift 25 & 26)

All are Welcome!