



The Hong Kong University of Science and Technology

Department of Mathematics

PhD THESIS EXAMINATION

**Variational Modeling and Numerical Simulations of
Active Thin Films and Evaporating Thin Films**

By

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ABSTRACT

Thin liquid films driven out of equilibrium exhibit rich dynamics due to the interplay of capillarity, viscous dissipation, and additional driving mechanisms such as internal activity or evaporation. This thesis develops a thermodynamically consistent framework based on Onsager's variational principle to model and simulate two representative systems: active nematic droplets and evaporating sessile droplets. For active nematic fluids, a long-wave model is derived in which activity competes directly with capillarity. Their balance defines a characteristic length scale that governs droplet shape and spreading behavior. Strong activity produces flattened steady droplets distinct from the classical parabolic profile. Asymptotic analysis and numerical simulations further reveal capillarity-dominated and activity-dominated spreading regimes, as well as a crossover in between. Spatial variations in wettability or activity can also drive directed droplet motion. For evaporating droplets, the model couples thin-film flow with vapor diffusion and interfacial evaporation. An intrinsic length scale separates diffusion-limited and transition-limited regimes. To resolve the evaporation singularity near the contact line, a coupled finite difference/boundary element method and a Lagrangian particle-transport approach are developed. Simulations reproduce non-uniform evaporation flux and characteristic deposition patterns associated with the coffee-ring effect. Overall, this thesis provides a thermodynamically consistent framework for analyzing thin-film flows in which internal activity or phase change dictates the interfacial dynamics.

Date : 16 June 2026, Tuesday

Time : 3:00 pm

Venue : Room 4472 (near Lifts 25/26)

Thesis Examination Committee:

- Chairman** : **Prof. Kristiaan NEYTS, ECE/HKUST**
- Thesis Supervisor** : **Prof. Tiezheng QIAN, MATH/HKUST**
- Member** : **Prof. Kun XU, MATH/HKUST**
- Member** : **Prof. Yang XIANG, MATH/HKUST**
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