The Hong Kong University of Science and Technology

Department of Mathematics

MPhil THESIS EXAMINATION

A Structure-Preserving Method for the Traveltime Gradient of the Eikonal Equation

By

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ABSTRACT

We propose a structure-preserving numerical method for a vector-valued partial differential equation whose solution describes the normalized traveltime gradient (orientation field) associated with the eikonal equation. Instead of first solving the scalar eikonal equation and then recovering the gradient by post-processing, our approach evolves the direction field directly. This direction field satisfies a nonlinear transport equation with values constrained to the unit sphere. The proposed scheme combines a Godunov-type upwind discretization with the analytical derivative of spherical linear interpolation (SLERP). The resulting tangent velocity is then advanced using the spherical forward Euler (SFE) method. Through this construction, the unit-sphere constraint is preserved at the discrete level, so the numerical iterates remain on $\mathbb{S}^2$ without additional projection or renormalization steps. Numerical experiments in both two and three dimensions show first-order convergence and verify the geometric consistency of the method. These results extend the classical fast sweeping idea to spherical-valued formulations and provide an efficient way to compute direction fields arising from wave propagation, geometric optics, and adjoint-based inverse problems.

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Time : 2:00 pm

Venue : Room 4502 (Lifts 25-26)

Thesis Examination Committee

Chairman	**:**	**Prof. Yang XIANG, MATH /HKUST**
Thesis Supervisor	**:**	**Prof. Shing Yu LEUNG, MATH/HKUST**
Member	**:**	**Prof. Zhichao PENG, MATH/HKUST**

(Open to all faculty and students)

The student's thesis is now being displayed on the reception counter in the General Administration Office (Room 3461).