



THE HONG KONG UNIVERSITY OF SCIENCE & TECHNOLOGY

Department of Mathematics

SEMINAR ON APPLIED MATHEMATICS

Fast Fourier spectral method for kinetic equations: from particle to wave

By

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Abstract

In this talk, I will present our recent progress on developing fast Fourier spectral methods for both classical kinetic equations and wave kinetic equations (WKE). In the first part, we focus on the Boltzmann equation, a fundamental model bridging many-particle system with macroscopic fluid dynamics. We introduce an efficient spectral formulation that approximates the non-cutoff Boltzmann collision operator and accelerates its computation through fast Fourier transforms (FFT), achieving accuracy and efficiency comparable to the cutoff case. Despite the practical success of the approaches, stability remains a significant challenge due to the lack of positivity and conservation. To address this, we develop a new analysis framework establishing spectral accuracy via a refined L^2 estimate of the negative part of the numerical solution. In the second part, we extend the fast spectral methodology to the WKE, the core equation in wave turbulence theory. By reformulating the high-dimensional nonlinear wave kinetic operator into a spherical integral — mirroring the structure of the Boltzmann operator — we reveal a double-convolution structure in Fourier space. This structure enables an efficient FFT-based evaluation of the wave interaction integral. Throughout the talk, I will demonstrate the accuracy, efficiency, and robustness of the proposed methods through a variety of 2D and 3D numerical experiments, and I will highlight several interesting phenomena and conjectures that emerge from these simulations.

Date : 17 June 2026 (Wednesday)
Time : 11:00a.m.-12:00noon
Venue : Room 4475 (Lift 25 & 26)

All are Welcome!