



THE HONG KONG UNIVERSITY OF SCIENCE & TECHNOLOGY

Department of Mathematics

SEMINAR ON APPLIED MATHEMATICS

Geometric Quasi-Linearization (GQL) and Structure-Preserving Analysis: From a General Framework to Unveiling Algebraic–Differential Relations in MHD

By

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Abstract

Preserving invariant regions (e.g., positivity of density and pressure, and the subluminal-velocity constraint in relativity) is fundamental to the physical consistency and mathematical wellposedness of hyperbolic conservation laws. However, developing high-order schemes that strictly respect nonlinear constraints remains a significant challenge for hyperbolic systems and computational fluid dynamics.

In the first part of this talk, we introduce the Geometric Quasi-Linearization (GQL) framework. Building on key insights from convex geometry, GQL transforms arbitrary nonlinear convex constraints into equivalent linear constraints by introducing free auxiliary variables. Crucially, GQL reveals a geometric insight: nonlinear constraints in the physical space are, in essence, linear when lifted to a higher-dimensional space. We establish the theoretical foundation of this framework and propose three effective construction methods, providing a unified approach to nonlinear invariant-domain-preserving analysis and design.

In the second part, we apply GQL to the structure-preserving analysis of (ideal/relativistic) compressible magnetohydrodynamics (MHD), unveiling an intrinsic algebraic–differential coupling between thermodynamic constraints (pressure positivity) and the involution constraint (divergence-free magnetic field) at both the numerical and PDE levels. We prove that the preservation of algebraic bounds is generally conditioned on the discrete divergence-free structure. By deriving the exact compatibility condition, we construct high-order schemes that achieve synergistic preservation of both algebraic and differential constraints. The robustness of these methods is demonstrated through extreme simulations, including low plasma beta ($\approx 10^{-10}$) blast waves and astrophysical jets with Mach numbers up to 106.

This talk is based on joint work with Prof. Chi-Wang Shu (Brown University), Prof. Huazhong Tang (Peking University), Prof. R’emi Abgrall (University of Zurich), and (current and former) students and postdocs of my research group, including Dr. Huihui Cao, Dr. Shengrong Ding, Dr. Haili Jiang, Dr. Mengqing Liu, Mr. Dongwen Pang, Ms. Manting Peng, Mr. Linfeng Xu, and Dr. Caiyou Yuan.

Date : 29 July 2026 (Wednesday)

Time : 4:00p.m. – 5:00p.m.

Venue : Room 5402 (near Lift 17/18)

All are Welcome!