The Hong Kong University of Science and Technology

Department of Mathematics

PhD THESIS EXAMINATION

Neural Network Methods for Nonparametric Regression

By

Mr Bokai YAN

ABSTRACT

This thesis develops approximation and statistical theory for neural network methods in nonparametric regression, with a focus on how network architecture, temporal dependence, target smoothness, and computational constraints affect prediction accuracy. First, we study deep ReLU recurrent neural networks. We prove that a carefully constructed recurrent neural network can simultaneously approximate a sequence of past-dependent Hölder functions at nearly optimal rates. We then apply these approximation results to derive non-asymptotic upper bounds for least squares empirical risk minimizers. Under both exponentially β-mixing and i.i.d. assumptions, the resulting estimators achieve the minimax optimal rate up to logarithmic factors. Second, we study softmax-activated Transformer networks. For Hölder sequence-to-sequence target functions on matrix domains, we establish explicit L^p-approximation bounds for all $p \in [1, \infty]$ and derive excess risk bounds for empirical risk minimizers with β-mixing covariates. Our analysis also clarifies the role of attention and allows extensions beyond softmax activation. Third, we propose a computable random feature estimator based on a two-layer ReLU network. The method fixes inner weights sampled from the unit sphere and optimizes only the outer coefficients by projected gradient descent, thereby avoiding the nonconvex joint training of all network parameters. The estimator attains nearly optimal rates for both Hölder targets and targets in the ReLU variation space. Together, these contributions advance neural network theory and methodology for nonparametric regression, with broader implications for machine learning and statistics.

Date : 7 August 2026, Friday

Time : 10:00 am

Venue : Room 5510 (Lifts 25-26), Academic Building, HKUST

***Thesis Examination Committee*:**

Chairman	**:**	**Prof. Daniel PEREZ PALOMAR, ECE/HKUST**
Thesis Supervisor	**:**	**Prof. Jianfeng CAI, MATH/HKUST** *(via online mode)*
Member	**:**	**Prof. Dong XIA, MATH/HKUST**
Member	**:**	**Prof. Rong TANG, MATH/HKUST**
Member	**:**	**Prof. Haibin SU, CHEM/HKUST**
External Examiner	**:**	**Prof. Zhentao SHI, Department of Economics/ The Chinese University of Hong Kong**

(Open to all faculty and students)

The student's thesis is now being displayed on the reception counter in the General Administration Office (Room 3461).