



**The Hong Kong University of Science and Technology**

**Department of Mathematics**

**PhD THESIS EXAMINATION**

**The Spin Chain Models and  $q$ -Character Theory via  
Semi-infinite Cohomology**

*By*

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**ABSTRACT**

In this thesis, we review the foundations of semi-infinite cohomology for a quasi-finite  $\mathbb{Z}$ -graded Lie algebra  $\mathfrak{g}$  and the relevant results in the work of Frenkel and Zeitlin. We then establish the  $\mathcal{U}_q(\mathfrak{sl}_2)$ -module and  $\mathcal{U}_q(\widehat{\mathfrak{sl}}_2)$ -module isomorphisms connecting the 0-th semi-infinite cohomology of the Virasoro algebra **Vir** to the representation theory of quantum group  $\mathcal{U}_q(\mathfrak{sl}_2)$  and quantum affine algebra  $\mathcal{U}_q(\widehat{\mathfrak{sl}}_2)$ . Based on these isomorphisms, we further relate the semi-infinite cohomology to the Heisenberg spin chain models and  $q$ -character theory.

**Date : 7 August 2026, Friday**

**Time : 10:00 am**

**Venue : Room 4472 (near Lifts 25/26)**

**Thesis Examination Committee:**

|                          |          |                                                                                                                  |
|--------------------------|----------|------------------------------------------------------------------------------------------------------------------|
| <b>Chairman</b>          | <b>:</b> | <b>Prof. Sen LI, CIVL/HKUST</b>                                                                                  |
| <b>Thesis Supervisor</b> | <b>:</b> | <b>Prof. Ivan Chi Ho IP, MATH/HKUST</b>                                                                          |
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| <b>Member</b>            | <b>:</b> | <b>Prof. Adrian PO, PHYS/HKUST</b>                                                                               |
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*(Open to all faculty and students)*

The student's thesis is now being displayed on the reception counter in the General Administration Office (Room 3461).