The Hong Kong University of Science and Technology

Department of Mathematics

MPhil THESIS EXAMINATION

Understanding Contrastive Learning: Theoretical Perspectives and an Analysis of Bilinear Matrix Representations

By

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ABSTRACT

Contrastive self-supervised learning has become a prominent paradigm for unsupervised representation learning. Its intuition is simple—pull positive pairs together and push negative pairs apart—yet explaining why the resulting representations perform well on downstream tasks remains a central theoretical question. After introducing the historical development and standard algorithmic frameworks of contrastive learning, this thesis examines several theoretical explanations for its success. It first studies latent-structure perspectives that connect contrastive objectives to downstream linear classification. Since latent-variable assumptions can be restrictive in augmentation-based learning, it then turns to theories that explain how the augmentation mechanism shapes the structure learned by the representation. This progression motivates a more concrete population-level question: what class-discriminative information can the contrastive loss preserve?

To study this question in a setting where the relevant structure can be characterised explicitly, we consider shared-covariance matrix Gaussian mixtures, an augmentation-enabled positive-pair distribution, and bilinear representations of the form

$$X \mapsto U^T X V.$$

Our main question is whether population InfoNCE preserves all row and column directions required for class discrimination. We first show that directions outside the Matrix Fisher row and column spaces can be removed without increasing the population objective, so class-irrelevant directions do not benefit the contrastive criterion. We then establish that, in the small-β regime and under additional structural assumptions, limiting population minimisers recover the full Matrix Fisher row and column spaces. Consequently, these limiting representations lose no class-discriminative direction and retain the information required for optimal linear classification in this model.

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Time : 4:00 pm

Venue : Room 2504 (Lifts 25-26)

Thesis Examination Committee

Chairman	**:**	**Prof. Rong TANG, MATH /HKUST**
Thesis Supervisor	**:**	**Prof. Dong XIA, MATH/HKUST**
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(Open to all faculty and students)

The student's thesis is now being displayed on the reception counter in the General Administration Office (Room 3461).