The Hong Kong University of Science and Technology

Department of Mathematics

MPhil THESIS EXAMINATION

A Human–AI Research Framework for Open Mathematical Problems: Design and Application to Erdős Problem 354

By

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ABSTRACT

Open mathematical research requires useful representations and intermediate claims, as well as a verified final proof. This thesis investigates how a human researcher can organise artificial intelligence to support that process while retaining mathematical control and reliable evidence of progress. A critical literature review compares discovery systems, formal provers and research agents, and derives design requirements concerning feedback, research memory, human decisions and mathematical understanding.

The thesis proposes and designs a human–AI research framework centred on audit and revision, systematising collaborative mechanisms used during the investigation. Its state model records versioned proof obligations, dependencies, counterexamples, decisions and verification artifacts. Proposer/reviser, auditor and formalizer roles use separate agents with controlled task packages and private memories. Independent AI sessions were used in the research. Human judgement directs the target and strategy and adjudicates disputed claims; deterministic computation and formal checking support acceptance.

The author-led investigation using these mechanisms resolves the base-2 completeness question in Erdős Problem 354. For positive real parameters with irrational ratio, the nonzero values of the two dyadic floor sequences are proved strongly complete: after any finite deletion, every sufficiently large integer is a sum of distinct remaining values. The proof connects a finite certificate to persistent integer meshes, permanent descent and rational-approximation windows, and is formalized in Lean.

Evaluation reproduces five finite-check suites and twelve audit-software tests, relates released sources to archived formal-verification reports, and analyses documented revisions. The case establishes a successful application and reproducible verification components; comparative efficiency and general open-problem success rates remain unmeasured. The methodology connects human direction and controlled AI collaboration to verifiable and interpretable mathematical outcomes.

Date : 4 December 2026, Friday
Time : 09:30 am
Venue : Room 4472 (Lifts 25-26)

Thesis Examination Committee

Chairman	**:**	**Prof. Frederick Tsz-Ho FONG, MATH /HKUST**
Thesis Supervisor	**:**	**Prof. Kani CHEN, MATH/HKUST**
Member	**:**	**Prof. Maosheng XIONG, MATH/HKUST**

(Open to all faculty and students)

The student's thesis is now being displayed on the reception counter in the General Administration Office (Room 3461).