Workshop on Inverse Problems and Scientific Computing

Date: Jan 3-5, 2025

Organizing Committee:
Hai Zhang (chair), Faouzi Triki, Gary Hu, Xiang Xu, Junshan Lin, Peijun Li, Lai Jun

Venue: Lecture Theatre G, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong S.A.R., China.

Sponsor: Department of Mathematics, The Hong Kong University of Science and Technology

Overview: Inverse problems involve estimating the causes behind specific observations or measurements, which are often ill-posed. These problems have far-reaching applications in various fields, including geophysics, medical imaging, and materials science. The primary objective of this workshop is to convene a gathering of international experts and researchers in the fields of inverse problems and scientific computing to share and discuss the latest developments and advancements. The workshop will provide a platform for in-depth discussions on recent breakthroughs, challenges, and applications of inverse problem techniques, encompassing theoretical foundations and computational methods. The goal is to foster collaborations, facilitate the exchange of ideas, and promote innovative solutions to the complex challenges posed by inverse problems.

Registration： Jan 3, 2025 (Friday)

Schedule of Jan 4 (Saturday)

9:00 AM - 9:10 AM Opening Remarks
9:10 AM - 9:40 AM Prof. Haijun Wu
9:40 AM - 10:10 AM Prof. Yin Tao

10:10 AM – 10:40 AM Coffee Break

10:40 AM - 11:10 AM Prof. Shuai Lu
11:10 AM - 11:40 PM Dr. Yaohua Zang

11:40 AM - 12:10 PM Prof. Yixian Gao

12:10 PM - 2:00PM Lunch Break (UC Bistro)

2:00 PM - 2:30 PM Prof. KiHyun Yun

2:30 PM - 3:00 PM Prof. Lei Zhang

3:00 PM - 3:30 PM Prof. Dong Wang

3:30 PM - 4:00 PM Coffee Break

4:00 PM - 4:40 PM Keynote Talk by Prof. Hyeonbae Kang
4:40 PM - 5:10 PM Prof. Yuliang Wang
5:10PM- 5:40PM Prof. Junliang Lyu

6:30 PM - 8:30 PM Banquet

Schedule of Jan 5 (Sunday)

9:00 AM - 9:40 AM Keynote Talk by Prof. Schotland
9:40 AM - 10:10 AM Prof. Wangtao Lu

10:10 AM – 1O:30 AM Coffee Break

10:30 AM - 10:55 AM Prof. Yiwen Lin

10:55AM - 11:20 AM Prof. Yue Zhao

11:20 AM – 11:45AM Prof. Ping Liu
11:45 AM- 12:10PM Prof. Xiaokai Yuan

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12: 10PM-2PM. Lunch at China Garden

2:00 PM – 5:00PM Free discussion

Talk title and Abstract

Professor Yixian Gao

Affiliation: Northeast Normal University

Title：Analysis of Subwavelength Resonances in High Contrast Elastic Media

Abstract：In this talk, we are devoted to the mathematical study of the wave scattering by a hard elastic obstacle embedded in a soft elastic body in three-dimensional space. We characterize subwavelength resonances by the fact that the determinant of a matrix vanishes, and present the asymptotic expansions of the subwavelength resonant frequencies in the low-frequency region . We give the representation of the scattered field in the interior domain, and establish the transversal and longitudinal far-field patterns for the scattered field the exterior domain.

Professor Hyeonbae Kang

Affiliation: Inha University

Title: Neutral inclusions, an over-determined problem for confocal ellipsoids, and more

Abstract: An inclusion is said to be neutral to uniform fields if upon insertion into a homogeneous medium with a uniform field it does not perturb the uniform field at all. It is said to be weakly neutral if it perturbs the uniform field mildly. Such inclusions are of interest in relation to invisibility cloaking and effective medium theory. There have been some attempts lately to construct or to show existence of such inclusions in the form of core-shell structure or a single inclusion with the imperfect bonding parameter attached to its boundary. In this talk we review recent progress in such attempts. We also discuss the over-determined problem for confocal ellipsoids which is closely related with the neutral inclusion problem, and its equivalent formulation in terms of Newtonian potentials. We will also talk about recent applications to stress estimate on a biological body.

Professor Yiwen Lin

Affiliation: Shanghai Jiao Tong University

Title: Recent studies on random scattering problems and multiscale kinetic modelling with uncertainties

In this talk, I will present the research I have conducted in recent years on random scattering problems and multiscale kinetic modelling. In addressing the challenges of wave propagation with uncertainties, I proposed an MCCUQ reconstruction method for reconstructing key statistics of the unknown random structures and established the first a priori bound explicitly with wavenumbers for random surface scattering problems. Additionally, due to the technical bottleneck in engineering experiments that makes it difficult to distinguish droplet sizes, I incorporated particle size into traditional two-phase flow, developing a kinetic-fluid coupled model system with randomness, which has been validated both theoretically and numerically. These are joint work with Gang Bao (ZJU), Xiang Xu (ZJU) and Shi Jin (SJTU).

Professor Ping Liu

Affiliation: Zhejiang University

Title: A mathematical theory of computational resolution limit and super-resolution

Abstract: Due to the physical nature of wave propagation and diffraction, there is a fundamental diffraction barrier in optical imaging systems which is called the diffraction limit or resolution limit. Rayleigh investigated this problem and formulated the well-known Rayleigh limit. However, the Rayleigh limit is empirical and only considers the resolving ability of the human visual system. On the other hand, resolving sources separated below the Rayleigh limit to achieve so-called “super-resolution” has been demonstrated in many numerical experiments.

In this talk, we will propose a new concept “computational resolution limit” which reveals the fundamental limits in super-resolving the number and locations of point sources from a data-processing point of view. We will quantitatively characterize the computational resolution limits by the signal-to-noise ratio, the sparsity of sources, and the cutoff frequency of the imaging system. As a direct consequence, it is demonstrated that l_0 optimization achieves the optimal order resolution in solving super-resolution problems. The generalization and application of our results will be introduced as well.

Professor Shuai Lu,

Affiliation: Fudan University

Title: Function and derivative approximation by shallow neural networks

Abstract: We investigate a Tikhonov regularization scheme specifically tailored for shallow neural networks within the context of solving a classic inverse problem: approximating an unknown function and its derivatives within a unit cubic domain based on noisy measurements. The proposed Tikhonov regularization scheme incorporates a penalty term that takes three distinct yet intricately related network (semi)norms: the extended Barron norm, the variation norm, and the Radon-BV seminorm. These choices of the penalty term are contingent upon the specific architecture of the neural network being utilized. We establish the connection between various network norms and particularly trace the dependence of the dimensionality index, aiming to deepen our understanding of how these norms interplay with each other. We revisit the universality of function approximation through various norms, establish rigorous error-bound analysis for the Tikhonov regularization scheme, and explicitly elucidate the dependency of the dimensionality index, providing a clearer understanding of how the dimensionality affects the approximation performance and how one designs a neural network with diverse approximating tasks. It is a joint work with Yuanyuan Li (Fudan University).

Professor Wangtao Lu

Affiliation: Zhejiang University

Title: An FFT accelerated PML-BIE Solver for Three-Dimensional Acoustic Wave Scattering in Layered Media

Abstract: This paper is concerned with three-dimensional acoustic wave scattering in two-layer media, where the two homogeneous layers are separated by a locally perturbed plane featuring an axially symmetric perturbation. A fast boundary integral equation (BIE) method is proposed to solve the scattering problem within a cylindrical perfectly matched layer (PML) truncation. We use PML-transformed Green's functions to derive BIEs in terms of single- and double-layer potentials for the wavefield and its normal derivative on the boundary of each truncated homogeneous region. These BIEs, combined with interface and PML boundary

conditions, form a complete system that accurately approximates the scattering problem. An FFT-based approach is introduced to efficiently and accurately discretize the surface integral operators in the BIEs, where a new kernel splitting technique is developed to resolve instabilities arising from the complex arguments in Green's functions. Numerical experiments demonstrate the efficiency and accuracy of the proposed method, as well as the exponential decay of truncation errors introduced by the PML.

Professor Junliang Lyu
Affiliation: Jilin University

Title：Machine learning methods for scattering and inverse scattering problems

Abstract：In this talk, I will describe the latest progress we have made in solving scattering problems and inverse scattering problems. For the unbounded domain scattering problem, we have designed an alternatively-optimized SNN, which is fast and accurate. Meanwhile, we design a machine learning strategy that can solve the multi-body scattering problem without prior knowledge of the number of obstacle bodies.

Professor John C Schotland
Affiliation: Yale University

Title: Inverse Born Series for Nonlinear and Nonlocal PDEs

Abstract: The inverse Born series (IBS) is a direct reconstruction method that has been applied to inverse scattering problems in a variety of physical settings. In this talk I will discuss recent work in quantum and nonlinear optics involving the IBS for nonlocal and nonlinear PDEs.

Professor Yin Tao
Affiliation: Chinese Academic of Science, School of Mathematics and System Sciences

Title: CAD-CAE: boundary integral equation method

Abstract: This talk will present some recent progresses on the fast and high-order boundary integral equation solvers for solving the frequency-domain wave scattering problems. Some

regularization techniques and the associated theoretical analysis of the singular integral operators are developed to reduce the singular integrals to combinations of weakly-singular integral operators and surface differential operators whose numerical evaluations can be achieved by means of a newly developed Chebyshev-based rectangular-polar solver. This provides a novel CAD-CAE approach which can avoid the evaluation of computational mesh and can efficiently treat different complex CAD geometries. Numerical examples will be presented to show the accuracy and efficiency of the proposed solvers.

Professor Dong Wang
Affiliation: The Chinese University of Hong Kong, Shenzhen

Title: Weak Adversarial Networks for Constrained optimization with applications in solving PDE and topology optimization

Abstract: In this talk, we integrate the networks and adversarial training into constrained optimization problems to develop a framework algorithm for constrained optimization problems. For such problems, we first transform them into minimax problems using the augmented Lagrangian method and then use two (or several) deep neural networks(DNNs) to represent the primal and dual variables respectively. The parameters in the neural networks are then trained by an adversarial process. The proposed architecture is relatively insensitive to the scale of values of different constraints when compared to penalty based deep learning methods. Through this type of training, the constraints are imposed better based on the augmented Lagrangian multipliers. Extensive applications in solving partial differential equations, data driven system discovery, constrained optimization problems will be discussed.

Professor Yuliang Wang
Affiliation: Beijing Normal University)

Title: Near-field inverse obstacle scattering by flexural waves: method of transformed field expansion

Abstract: In this talk, we investigate the inverse scattering problem of an obstacle embedded in a thin plate using near-field measurements of flexural waves. The forward scattering problem is reduced to a coupled system of boundary value problems for the propagating and evanescent waves. Assuming the obstacle is a small perturbation of a circle, we employ the method of transformed field expansions to express the solution as a power series, obtaining closed-form expressions for the zeroth and first-order terms. These expressions are then used to derive an approximate reconstruction formula for the inverse scattering problem. We explore different types of incident fields, some of which lead to simplified and more efficient reconstruction methods. Numerical experiments demonstrate the effectiveness and efficiency of the proposed approach.

Professor Haijun Wu

Affiliation: Nanjing University

Title: Adaptive FEM for Helmholtz Equation with Large Wavenumber

Abstract: A posteriori upper and lower bounds are derived for the linear finite element method (FEM) for the Helmholtz equation with large wavenumber. It is proved rigorously that the standard residual type error estimator seriously underestimates the true error of the FE solution for the mesh size h in the pre-asymptotic regime, which is first observed by Babuska et. al. in 1997 for a one dimensional problem. By establishing an equivalence relationship between the error estimators for the FE solution and the corresponding elliptic projection of the exact solution, an adaptive algorithm is proposed and its convergence and quasi-optimality are proved under the condition that $k^3h_0^{1+\alpha}$ is sufficiently small, where k is the wavenumber, h_0 is the initial mesh size, and $\frac12<\alpha\le 1$ is a regularity constant depending on the maximum reentrant angle of the domain. Numerical tests are given to verify the theoretical findings and to show that the adaptive continuous interior penalty finite element method (CIP-FEM) with appropriately selected penalty parameters can greatly reduce the pollution error and hence the residual type error estimator for this CIP-FEM is reliable and efficient even in the pre-asymptotic regime.

Professor Xiaokai Yuan

Affiliation: Jilin University

Title: Convergence of the PML method for the biharmonic wave scattering problem in periodic structures

Abstract: This talk investigates the scattering of biharmonic waves by a one-dimensional periodic array of cavities embedded in an infinite elastic thin plate. The transparent boundary conditions are introduced to formulate the problem from an unbounded domain to a bounded one. The well-posedness of the associated variational problem is demonstrated utilizing the Fredholm alternative theorem. The perfectly matched layer (PML) method is employed to reformulate the original scattering problem, transforming it from an unbounded domain to a bounded one. The transparent boundary conditions for the PML problem are deduced, and the well-posedness of its variational problem is established. Moreover, exponential convergence is achieved between the solution of the PML problem and that of the original scattering problem.

Professor KiHyun Yun

Affiliation: Hankuk University of Foreign Studies

Title: Grain Boundary Evolution in 3D

Abstract: In this talk, we consider a network of grains separated by interfaces and facets, which evolve according to curvature. These facets intersect along curves that converge at specific points. Such systems are governed by nonlinear partial differential equations with corresponding boundary conditions. We investigate the local-in-time existence of solutions for the simplest configuration near an equilibrium state.

Dr. Yaohua Zang

Affiliation: Technical University of Munich, Germany

Title: ParticleWNN: a Weak-from Deep Learning Framework for Solving Partial Differential Equations and Inverse Problems

Abstract: In this presentation, we introduce ParticleWNN, an innovative neural network framework for solving challenging partial differential equations (PDEs) and related inverse problems based on the weak form. Our approach parameterizes the solution with deep neural networks (DNNs) and employs compactly supported test functions within small, localized regions, significantly reducing computational cost while enhancing accuracy compared to other deep learning-based methods such as vanilla PINNs and VPINNs. ParticleWNN

demonstrates exceptional capability in capturing sharp gradients and handling complex geometries, making it well-suited for nonlinear problems, high-dimensional settings, and inverse problems involving noisy data. We highlight the effectiveness of our method through diverse applications, including the Allen-Cahn equation, the Navier-Stokes equations, Stokes flow over irregular domains, and high-dimensional PDEs, emphasizing its robustness and versatility. This research is conducted in collaboration with Professor Gang Bao.

Professor Yue Zhao

Affiliation: Central China Normal University

Title: Stability for an inverse source problem with inhomogeneous damping

Abstract：In this talk, we present a stability estimate for an inverse source problem with inhomogeneous damping. In the stability, the dependence on the damping coefficient is traced. The scattering theory and semiclassical analysis are employed to derive an analytic domain and estimates for the resolvent of the elliptic operator.

Professor Lei Zhang

Affiliation: Zhejiang University of Technology

Title: Composite scattering and inverse scattering in a multilayered medium

Abstract: Scattering from the targets embedded in a multilayered medium has attracted much interest in recent years for extensive applications, such as sea radar target detection and underwater radar surveillance. Over the past few decades, the scattering of electromagnetic waves by various composite targets has been investigated extensively by many researchers. This talk concerns the mathematical analysis and numerical method for composite scattering and inverse scattering in a multilayered medium. The composite scattering problem from obstacles and the unbounded rough surface refers to the electromagnetic waves interacting with the interface of the medium and the obstacles in the medium. We will discuss the scattering problems and introduce the recent progress.