MATH 4033 Calculus on Manifolds

Course outline - Spring 2023-2024

1. Instructor :
   Prof. Guowu, MENG, mameng@ust.hk
   Office: Room 3484 (via lift 25-26)

1. Teaching Assistant
   YOU, Lei (lyou@connect.ust.hk)

2. Meeting Time and Venue
   Lecture: L1 Tue/Thur, 13:30—14:50, LTH
   Tutorial: T1A Fri, 18:00-18:50

Course Description: (3 Credits)

This course is designed to provide students with an introduction to smooth manifolds and differential geometry. Through rigorous study and application of fundamental concepts and techniques, students will develop a solid foundation in this branch of mathematics. The course will cover a range of topics, including smooth manifolds, tensor fields, differentiation, and integration. Emphasis will be placed on the calculus part.

Course Objectives:

- Develop a deep understanding of smooth manifolds and smooth maps.

- Explore the properties and applications of tensor fields and functions.

- Master various kinds of differentiation, including Lie differentiation, exterior differentiation, covariant differentiation, and the Schouten–Nijenhuis bracket.

- Gain proficiency in integration techniques and apply them to solve problems using Stokes's Theorem.

- Enhance problem-solving and critical thinking skills through the application of differential geometry concepts.

Prerequisite : Grade A- or above in MATH 2023 AND B- or above in MATH 2131

A solid understanding of linear algebra and multivariable calculus is essential for success in this course. Students should be familiar with concepts such as vectors, matrices, partial derivatives, and multiple integrals.

Textbook:

The primary reference for this course will be [Calculus III](#)

[Links to an external site.](#). Additional recommended resources will be provided throughout the course.

Schedule:

Classes will be held on Tuesdays and Thursdays from 13:30 to 14:50 am in Lecture Theater H, located in Lifts 25/26.

Grading Policy:

The final grade for this course will be based on the following components:

- Homework assignments: 30%

- Midterm exams: 30%

- Final exam: 40%

Course Topics:

0. Review of Linear Algebra and Multivariable Calculus

1. Smooth Manifolds and Smooth Maps

   - Definition and examples of smooth manifolds

   - Smooth maps and their properties

   - Tangent spaces and vector fields on manifolds

2. Tensor Fields and Functions

   - Introduction to tensor fields and their properties

   - Tensor products and contractions

   - Covariant and contravariant tensors

3. Differentiations

   - Lie differentiation and Lie brackets

   - Exterior differentiation and differential forms

   - Covariant differentiation and connections

   - The Schouten–Nijenhuis bracket and Multivecor fileds

4. Integration and Stokes's Theorem

   - Integration of differential forms

   - Stokes's Theorem and its applications

Note:

The schedule and topics are subject to change at the discretion of the instructor. Any changes will be communicated in advance.