# MATH 4221 Euclidean and Non-Euclidean Geometry

Course Outline - Spring 2023-2024

**1.Instructor**

Professor Zhu, Yongchang
Contact Details: Rm 3459; e-mail: mazhu@ust.hk
Office hour: Wed 4:30-5:30pm

**2.Teaching Assistant**

Tan, Hongze
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**3.Meeting Time and Venue**

*Lectures*
**Date/Time:** Monday and Wednesday, 12:00-13:20
**Venue**: 2504

*Tutorials*
**Date/Time, Venue:** T1a, Thursday, 9:30-10:20; 2463

**4.Course Description**

Duration: one semester. Credits: 3 units.
Pre-requisite: Math 2033 or Math 2043 or Math 2111 or Math 2121 or Math 2131 or Math 2350

The course is intended for senior undergraduate math students and has two primary objectives. Firstly, it aims to introduce students to both Euclidean and Non-Euclidean geometry using axioms and emphasize the significance of axiomatic methods in mathematics. Secondly, the course will focus on the study of plane hyperbolic geometry using Poincare's model and introduce the Mobius transformation group to showcase the role of symmetry in geometry.

**5. Intended Learning Outcomes**

Upon successful completion of this course, students should be able to:

1 Understand the importance of axiomatic method.
2. Recognize and use appropriately important concepts and definitions in geometry and know how to study geometric problems using groups and symmetry.
3. Understand how the mathematical knowledge students learnt before can be better organized and generalized.

**6. Assessment Scheme**

a. Examination duration: 2 hours

b. Percentage of course work, examination

| Assessment: | | Assessing Course ILOs |
|---|---|---|
| Coursework: | 50% | 1, 2, 3, 4, |
| Examination: | 50% | 1, 3, 4 |

**7. Student Learning Resources**

Lecture Notes:
Lecture Notes can be downloaded at the course home page in Canvas.

Textbooks:

1. Paul Kelly, Gordon Matthews, The Non-Euclidean, Hyperbolic Plane. Springer-Verlag, 1981
2. James W. Anderson, Hyperbolic Geometry, 2nd edition, Springer-Verlag, 2005

**8. Teaching and Learning Activities**

Scheduled activities: 3 hrs (lecture)

**9.Course Schedule**

| Week | Content | Remarks |
|---|---|---|
| 1 | Historical background, axiomatic method, examples of axiomatic systems | |
| 2 | Hilbert Axioms. Birkhoff's axioms of absolute plane geometry, Euclidean geometry, hyperbolic geometry. Theorems implied by axioms | |
| 3 | Analytic model for Euclidean geometry | |
| 4 | Group SL(2,R) and Mobius transformation | |
| 5 | Euclidean length and hyperbolic length of paths | |
| 6 | Hyperbolic distance | |
| 7 | Poincare's upper half plane model for hyperbolic plane geometry I | |
| 8 | Poincare's upper half plane model for hyperbolic plane geometry II | Midterm Review |
| 9 | Unit Disc Model for Hyperbolic Plane Geometry | |
| 10 | Convexity, Hyperbolic polygons | |
| 11 | Gauss–Bonnet Formula and applications | |
| 12 | Trigonometry in the hyperbolic plane | |
| 13 | Higher dimensional hyperbolic spaces | |
| 14 | Introduction to Riemannian geometry | |