Math 2131 Honors in Linear and Abstract Algebra I
Course Outline - Fall 2024 - 2025

1. Instructor(s)

Name: Min Yan
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2. Teaching Assistant(s)

Name: Junwei Ma
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3. Meeting Time and Venue

Lecture: L1
Date/Time: Monday 15:00 - 16:20, Friday 10:30 - 11:50
Venue: Room 2404

Tutorial: T1A
Date/Time: Monday 19:00-19:50
Venue: Room 1104

Tutorial: T1B
Date/Time: Tuesday 11:00-11:50
Venue: LSK 1027

4. Course Description

Credit Points: 4

Pre-requisite: Grade A in AL Pure Mathematics; or grade A- or above in MATH 1014/ MATH1020/MATH1024

Exclusion: MATH2111, MATH2121, MATH2350

Brief Information/synopsis:
The MATH 2131 and 3131 is a sequence of rigorous introduction to linear algebra and abstract algebra. Vector spaces over the fields of real numbers and complex numbers, linear transformations, geometry, groups, bases, abstract fields, rings, change of bases, spectral theorems.

5. Intended Learning Outcomes

Upon successful completion of this course, students should be able to:
1. Understand and calculate the basic concepts of linear algebra, in the equivalent settings of systems of linear equations, linear combinations of vectors, and linear transformations.
2. Understand and calculate the most important topics in linear algebra, including orthogonality, determinant, and diagonalisation.
3. Able to do mathematics in rigorous, conceptual, and abstract way.
4. Apply linear algebra concepts to model, solve, and analyse real-world situations

6. Assessment Scheme

Composition:
20% homework (~10 assignments, ILO 1, 2, 3, 4)
30% midterm exam (3 hours, ILO 1, 2, 3, 4)
50% final exam (3 hours, ILO 1, 2, 3, 4)

Grading scheme:
A: 85 points, the student has fully mastered all the theory of concepts, and can do all the corresponding calculations.
B: 60 points, the student has mastered enough theory and concepts, but may have missed some more advanced parts, and can do most of the corresponding calculations.
C: 35 points, the student has mastered the minimum theory and concepts,, and can do minimum of the corresponding calculations.
D and F: the student does not understand some major parts, and cannot do most calculations. The line between D and F is whether the student shows enough effort. If the gap in grade from C is too much, or the student did not submit most of the homework, then he/she gets F. Otherwise the student gets D.

7. Student Learning Resources

Text(s): Linear Algebra, lecture note by the instructor.

8. Teaching and Learning Activities

Lecture:
Listen to the lectures delivered by the instructor. Learn the concepts through definitions, examples, and discussions.

Tutorial:
Learn how to solve problems. Instructor my provide additional material not covered in the lecture.

Online:
A WeChat group is set up, where the students, the instructor, and the TA can discuss and communicate all things about the course.

9. Course Schedule

1. *System of linear equations*: row operation, row echelon form, existence and uniqueness, rank
2. *Euclidean space*: Euclidean vector, dot product, span, linear independence, subspace of Euclidean space, basis
3. *Linear transformation*: linear transformation and its matrix, matrix operation, onto, one-to-one, inverse, LU-decomposition
4. *Vector space*: abstract vector space, linear combination (span, independence, basis), linear transformation (isomorphism, matrix with respect to basis, change of basis), sum and direct sum of subspaces, quotient space, dual space
5. *General linear algebra*: complex linear algebra, complexification, linear algebra over any field, polynomial, field extension, trisection of angle
6. *Orthogonality*: orthogonal projection, orthogonalisation, isometry, QR-decomposition, least square solution, orthogonal complement, orthogonal sum, inner product space, complex inner product
7. *Multilinearity*: tensor, dual pairing, algebra of determinant, geometry of determinant, exterior algebra
8. *Spectral theory*: eigenspace, diagonalisation, orthogonal diagonalisation of normal operator, singular value decomposition, nilpotent operator, Jordan canonical form, rational operator