MATH 4141 – Number Theory and Applications

Fall 2024 Course Syllabus

Lecture

Instructor:	**Prof. IP, Ivan Chi-Ho**
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Tutorial

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Meeting Time and Venue

Lecture:	Monday	15:00 – 16:20	LTK
	Friday	10:30 – 11:50	LTK
T1A:	Monday	19:00 – 19:50	Rm 1032, LSK Bldg

Course Description

This is a 4000-level course on Number Theory. It is designed simultaneously for pure math majors who are new to elementary number theory, as well as for advanced students who want in-depth knowledge about modern analytic approaches. The course is split into two parts.

- The first part covers the essentials of **elementary number theory**, including modular arithmetic, quadratic reciprocity, quadratic number fields, and properties of prime numbers.
- The second part covers the essentials of **analytic number theory**, including arithmetic functions, the proofs of the Prime Number Theorem and Dirichlet's Theorem on arithmetic progressions.

Students are expected to have general knowledge of linear algebra, mathematical analysis, as well as minimal abstract algebra (basics of group, ring, field) and complex analysis (e.g. Cauchy's integral formula).

Prerequisite:	MATH2131.
Co-requisite:	MATH3121.
Recommended also:	MATH2001, MATH2033, MATH4023
Credits:	3

Intended Learning Outcomes

By the end of this course, students should be able to:

1. Be familiar with the history of mathematical development through elementary number theory.
2. Be able to work with advanced methods in modern number theory.
3. Develop an understanding of the core ideas and concepts of analytic number theory for future studies in advanced number theory courses.

Assessment and Grading

The assessment consists of 5 Homeworks, Midterm and Final Examination.

Homework. There will be 5 homework sets during the semester consists of computation problems, short questions and long question. Students should submit each homework in form of a neatly written and scanned PDF, or a LaTeX-typed PDF on the Canvas system before the deadline. All the questions will be graded.

Examinations. There will be a 3-hour midterm exam during Week 6-8 (exact date to be confirmed), and a 3-hour final exam arranged by ARO.

	Weight	Assessment ILOs
Homework	20%	1,2,3
Midterm Examination	10–40%	1,2,3
Final Examination	40–70%	1,2,3

Grading Scheme. This course will be assessed using **criterion-referencing** and grades will **not** be assigned using a curve.

The total point is calculated by the following formula:

Total Point

$$= HW(20\%) + \begin{cases} \max\limits_{W\in[10,40]}(25, \text{Midterm} * W\% + \text{Final} * (80-W)\% & \text{if Midterm or Final} \geq 25 \\ \max(\text{Midterm}, \text{Final}) & \text{if Midterm and Final} < 25. \end{cases}$$

Scoring 25 points in an Exam will guarantee a 25% course total points. This will guarantee that students who performed satisfactorily in the Midterm and homeworks only can still pass the course with a maximum of a B- grade.

Final Grade Descriptors. Letter grades will be assigned depending on overall performance.
Obtaining a total point of 90% or above, or top 10%, will guarantee an A+.
Obtaining a total point of 70% or above will guarantee an A-range.
Obtaining a total point of 50% or above will guarantee a B-range.
Obtaining a total point of 30% or above will guarantee a passing grade.

Grades	Short Description	Elaboration on subject grading description
A	Excellent Performance	Demonstrates a comprehensive grasp of both elementary and analytic number theory, on both the conceptual understanding and computational aspects.
B	Good Performance	Shows good knowledge and understanding of both elementary and analytic number theory, but did not master the subject completely.
C	Satisfactory Performance	Possesses adequate knowledge of core subject matter, mostly on elementary number theory.
D	Marginal Pass	The student has threshold knowledge of elementary number theory only, and able to complete some of the easiest computations.
F	Fail	The student does not have sufficient understanding of even some fragments of topics, and is not even able to complete some of the easiest computations.

Academic Integrity

Students are expected to adhere to the university's academic integrity policy. Students are expected to uphold HKUST's Academic Honor Code and to maintain the highest standards of academic integrity. The University has zero tolerance of academic misconduct. Please refer to Academic Integrity — HKUST – Academic Registry for the University's definition of plagiarism and ways to avoid cheating and plagiarism.

Late submission Policy. To ensure fairness for students who submit assignments on time, a penalty for late submission is listed as follows, according to the timestamp of Canvas:

- Late submission between 0 to 24 hours, 50% penalty will be applied.
- Late submission for more than 24 hours will not be accepted.

Make-up Exam Policy. Make-up Midterm Exam will only be considered with at least a 4-hour notice and with proper support documents. Make-up Final will follow the make-up exam policy set by ARO.

Communication and Feedback. Assessment marks for individual assessed tasks will be communicated via Canvas within two weeks of submission. Feedback on assignments will include comments and corrections. Students who have further questions about the feedback including marks should consult the instructor or TA within five working days after the feedback is received through email or discord.

Course AI Policy. The use of Generative AI is permitted but students should make appropriate citation for its usage. Students should be critical of the response generated by AI and do not blindly copy the generated responses to your homework.

Student Learning Resources

Required Text: *Lecture Notes* by the instructor.

Lecture Slides and **Lecture Videos** will be available on canvas after class.

Additional References:

Elementary Number Theory:

1. *Number theory* by Robert Freud and Edit Gyarmati
2. *An introduction to the theory of numbers* by Ivan Niven, Herbert S. Zuckerman and Hugh L. Montgomery
3. *A course of elementary number theory* by Robert C. Vaughan.

Analytic Number Theory:

1. *Introduction to Analytic Number Theory* by Tom Apostol
2. *A Course in Arithmetic* by Jean-Pierre Serre.

Tentative Schedule

Lecture 1–5	Congruences
Lecture 6–8	Quadratic Reciprocity
Lecture 9–11	Quadratic Integers
Lecture 12–13	Prime Numbers
Lecture 14–17	Arithmetic Functions
Lecture 18–22	Prime Number Theorem
Lecture 23–25	Dirichlet's Theorem on Arithmetic Progression