

Math4033 Calculus on Manifolds
Course Outline- Spring 2025

1. Instructor(s)

Name: Guowu Meng

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2. Teaching Assistant(s)

Name: Jiantong Hou

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3. Meeting Time and Venue

Lectures:

Date/Time: Monday and Wednesday (9:00 – 10:20)

Venue: Classroom 4504, Lifts25/26

Tutorials:

Date/Time: Thursday (18:00-18:50)

Venue: Classroom 1407, Lifts25/26

4. Course Description

Credit Points: 4

Pre-requisite: Linear Algebra + Multivariate calculus

Exclusion: NIL

Brief Information/synopsis:

This course will cover the following topics:

- Smooth manifolds and smooth maps
- Tensor fields and functions
- Differentiation techniques, including Lie differentiation, exterior differentiation, covariant differentiation, and the Schouten–Nijenhuis bracket
- Integration and Stokes's Theorem

5. Intended Learning Outcomes

Upon successful completion of this course, students should be able to:

No.	ILOs
1	Understand the key definitions.
2	Know how to carry out local computations
3	Understand the connection and relevance to mechanics and electrodynamics
4	Solve typical problems in geometry and symmetry

6. Assessment Scheme

- a. Examination duration: 4.5 hrs
- b. Percentage of coursework, examination, etc.:

<u>Assessment</u>	<u>Assessing Course ILOs</u>
25% by coursework	1, 2, 3, 4
0% discussion report & participation	
75% by exam	1, 2, 3, 4

- c. The grading is assigned based on students' performance in assessment tasks/activities.

7. Student Learning Resources

Recommended Reading:

Text(s): Guowu Meng, "Calculus III", Lecture notes.

8. Teaching and Learning Activities

Scheduled activities: 4 hrs (lecture + tutorial)

9. Course Schedule

Keyword Syllabus:

- Linear Algebra: vector spaces and linear maps, dual vector spaces, tensors, bases, determinant.
- Affine Geometry: affine spaces and affine maps, Euclidean Spaces, rigid motions.
- Manifolds: smooth maps, parametrized smooth curves, velocity spaces, tangent spaces, cotangent spaces.
- Tensor fields: tangent bundle, cotangent bundle, tensor bundles, differential forms, multi-vector fields, metric tensor.
- Exterior Differentiation: meaning and characterization, application in physics.
- Lie differentiation: meaning and characterization, Cartan's Formula.
- Covariant Differentiation: meaning and characterization, connection with exterior differentiation, comparison with Lie differentiation.
- Schouten–Nijenhuis bracket: meaning and characterization, connection and comparison with exterior differentiation.
- Integration and Stokes's Theorem: definition of integration of differential forms, statement and proof of Stokes's theorem, comparison with integration theory in multivariate calculus.